H4 (left) and H4Φ (right) 24-Cell

The left images are scaled up to the same size as the right image in order to show more detail.

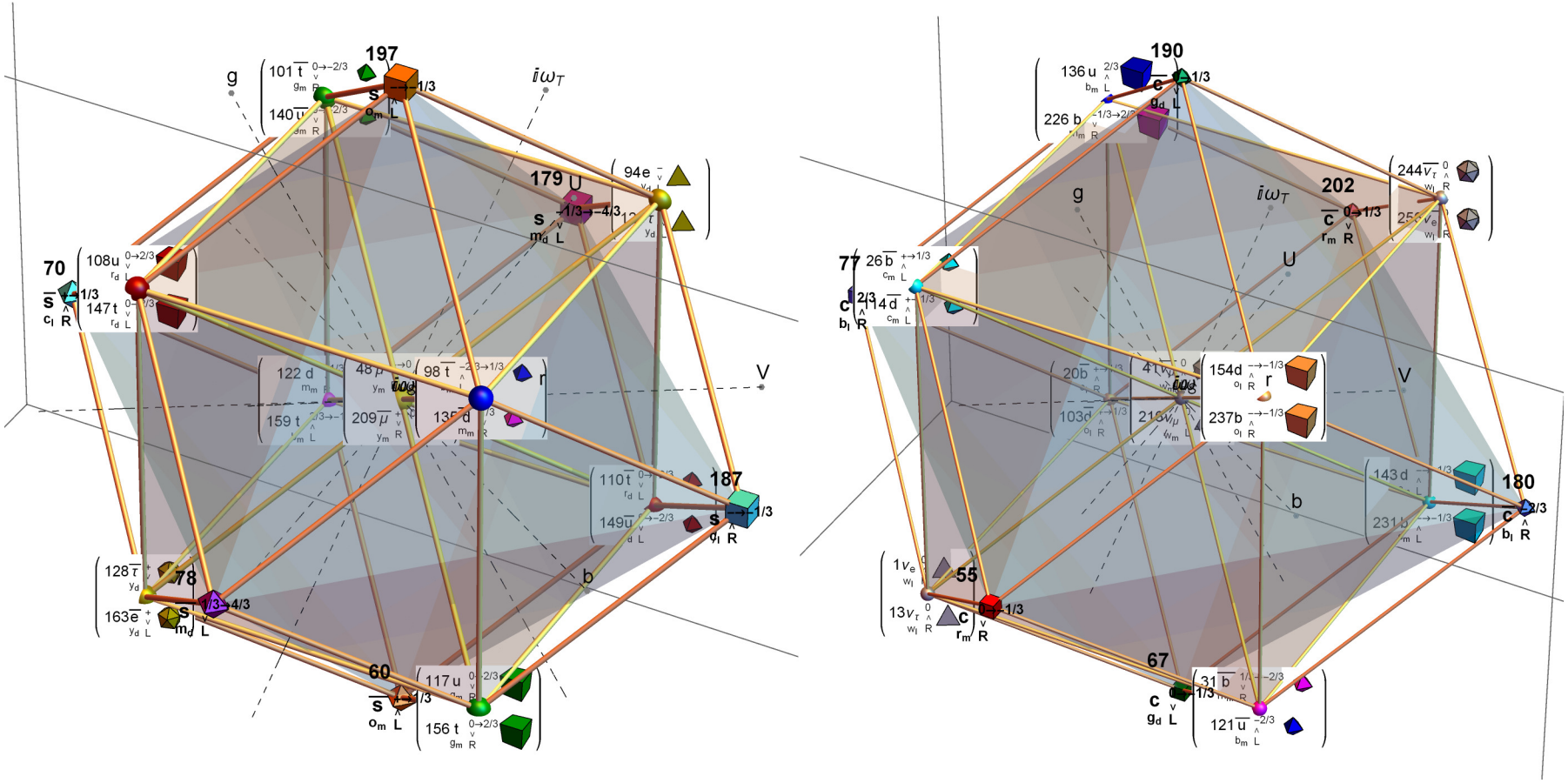
This accounts for the thicker edges and larger vertex shapes in the left image.

The light blue octahedron is the 8 vertex 16-cell.

The light red cube is the 16 vertex 8-cell (a Tesseract) with dual vertices at each corner.

In these 3D projections of hyper-dimensional objects, we do not scale the 4th dimension as is sometime shown for the Tesseract.

All objects (above and below) use the same projection basis vectors. The only difference is in the selection from the 240 8D E8 vertices being projected.



ColumnForm[MatrixForm[{Row@{Column[{#, pDisp@#, pLbl@#}, Center]} & /@ #}] & /@ {h4cell16, h4cell8, h4#cell16, h4#cell8}]

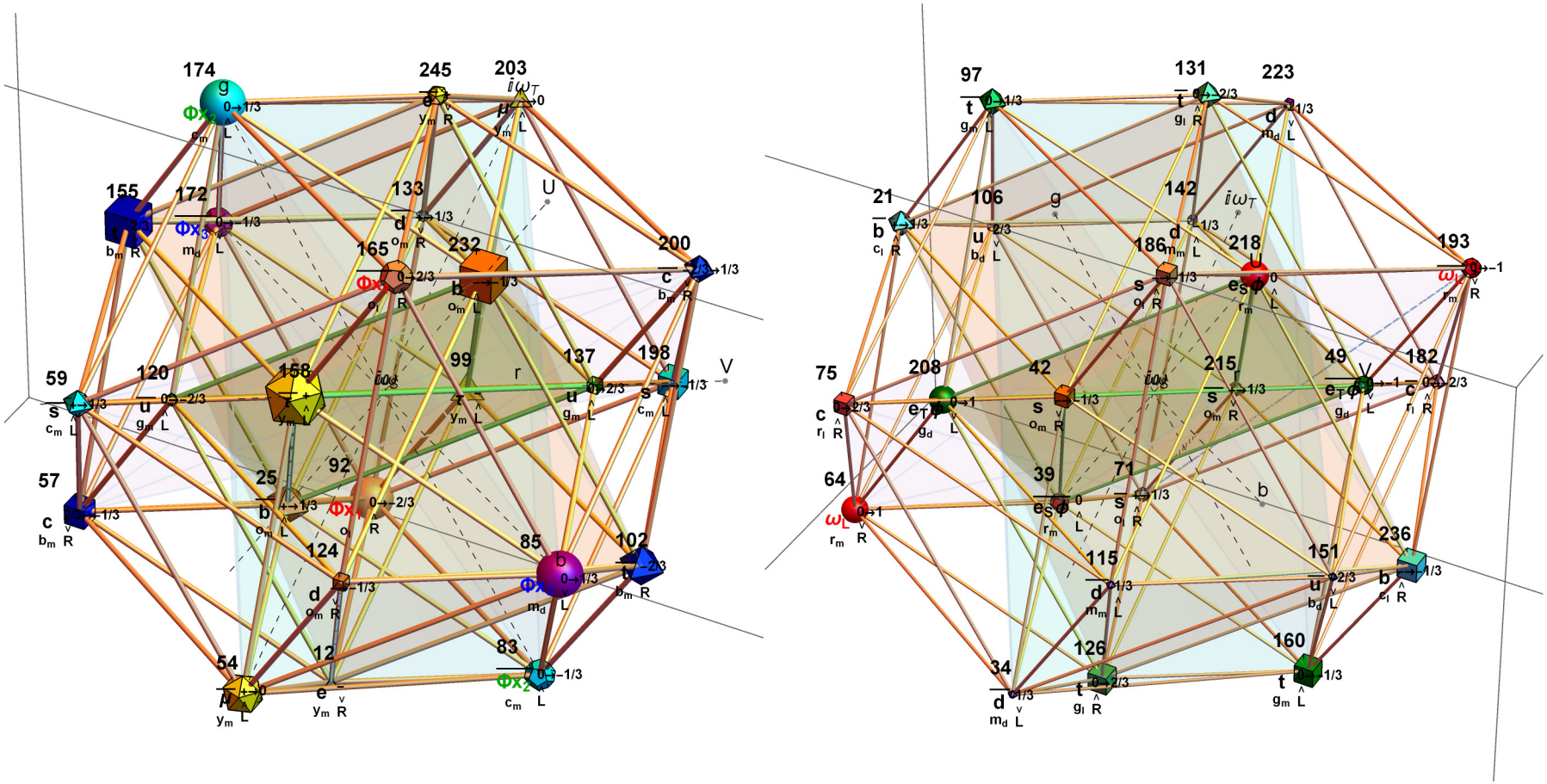
48	60	70	78	179	187	197	209
$\mu_{y_m}^{-\to 0}\ \vee\ R$	$\overline{s}_{o_m}^{+\to 1/3}\ \wedge\ L$	$\overline{s}_{c_l}^{+\to 1/3}\ \wedge\ R$	$\overline{s}_{m_d}^{1/3\to 4/3}\ \vee\ L$	$s_{m_d}^{-1/3\to -4/3}\ \vee\ L$	$s_{c_l}^{-\to -1/3}\ \wedge\ R$	$s_{o_m}^{-\to -1/3}\ \wedge\ L$	$\overline{\mu}_{y_m}^{+\to 0}\ \vee\ R$

94	98	101	108	110	117	122	128	129	135	140	147	149	156	159	163
$e_{y_d}^{-}\ \vee\ L$	$\overline{t}_{b_m}^{-2/3\to 1/3}\ \wedge\ L$	$\overline{t}_{g_m}^{0\to -2/3}\ \vee\ R$	$u_{r_d}^{0\to 2/3}\ \vee\ L$	$\overline{t}_{r_d}^{0\to -2/3}\ \vee\ L$	$u_{g_m}^{0\to 2/3}\ \vee\ R$	$d_{m_m}^{-1/3}\ \vee\ R$	$\overline{\tau}_{y_d}^{+}\ \vee\ L$	$\tau_{y_d}^{-}\ \vee\ L$	$\overline{d}_{m_m}^{1/3}\ \vee\ R$	$\overline{u}_{g_m}^{0\to -2/3}\ \vee\ R$	$t_{r_d}^{0\to 2/3}\ \vee\ L$	$\overline{u}_{r_d}^{0\to -2/3}\ \vee\ L$	$t_{g_m}^{0\to 2/3}\ \vee\ R$	$t_{b_m}^{2/3\to -1/3}\ \wedge\ L$	$\overline{e}_{y_d}^{+}\ \vee\ L$

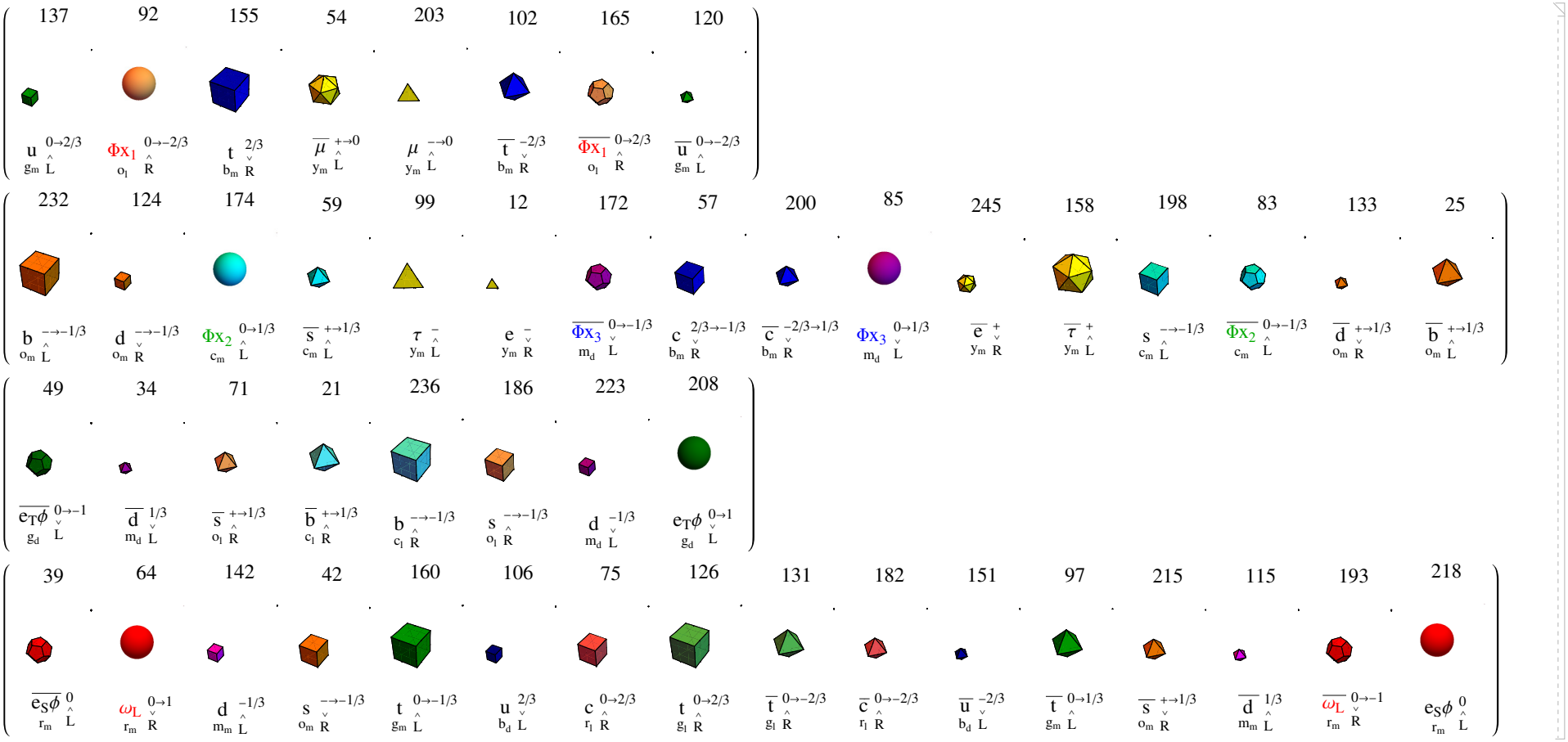
41	55	67	77	180	190	202	216
$\overline{\nu_\mu}_{w_m}^{0}\ \wedge\ L$	$c_{r_m}^{0\to -1/3}\ \vee\ R$	$c_{g_d}^{0\to -1/3}\ \vee\ L$	$c_{b_l}^{2/3}\ \wedge\ R$	$\overline{c}_{b_l}^{-2/3}\ \wedge\ R$	$\overline{c}_{g_d}^{0\to 1/3}\ \vee\ L$	$\overline{c}_{r_m}^{0\to 1/3}\ \vee\ R$	$\nu_\mu{}_{w_m}^{0}\ \wedge\ L$

1	13	20	26	31	103	114	121	136	143	154	226	231	237	244	256
$\nu_e{}_{w_l}^{0}\ \wedge\ R$	$\nu_\tau{}_{w_l}^{0}\ \wedge\ R$	$\overline{b}_{o_l}^{+\to 1/3}\ \wedge\ R$	$\overline{b}_{c_m}^{+\to 1/3}\ \wedge\ L$	$\overline{b}_{m_m}^{1/3\to -2/3}\ \vee\ R$	$\overline{d}_{o_l}^{+\to 1/3}\ \wedge\ R$	$\overline{d}_{c_m}^{+\to 1/3}\ \wedge\ L$	$\overline{u}_{b_m}^{-2/3}\ \wedge\ L$	$u_{b_m}^{2/3}\ \wedge\ L$	$d_{c_m}^{-\to -1/3}\ \wedge\ L$	$d_{o_l}^{-\to -1/3}\ \wedge\ R$	$b_{m_m}^{-1/3\to 2/3}\ \vee\ R$	$b_{c_m}^{-\to -1/3}\ \wedge\ L$	$b_{o_l}^{-\to -1/3}\ \wedge\ R$	$\overline{\nu_\tau}_{w_l}^{0}\ \wedge\ R$	$\overline{\nu_e}_{w_l}^{0}\ \wedge\ R$

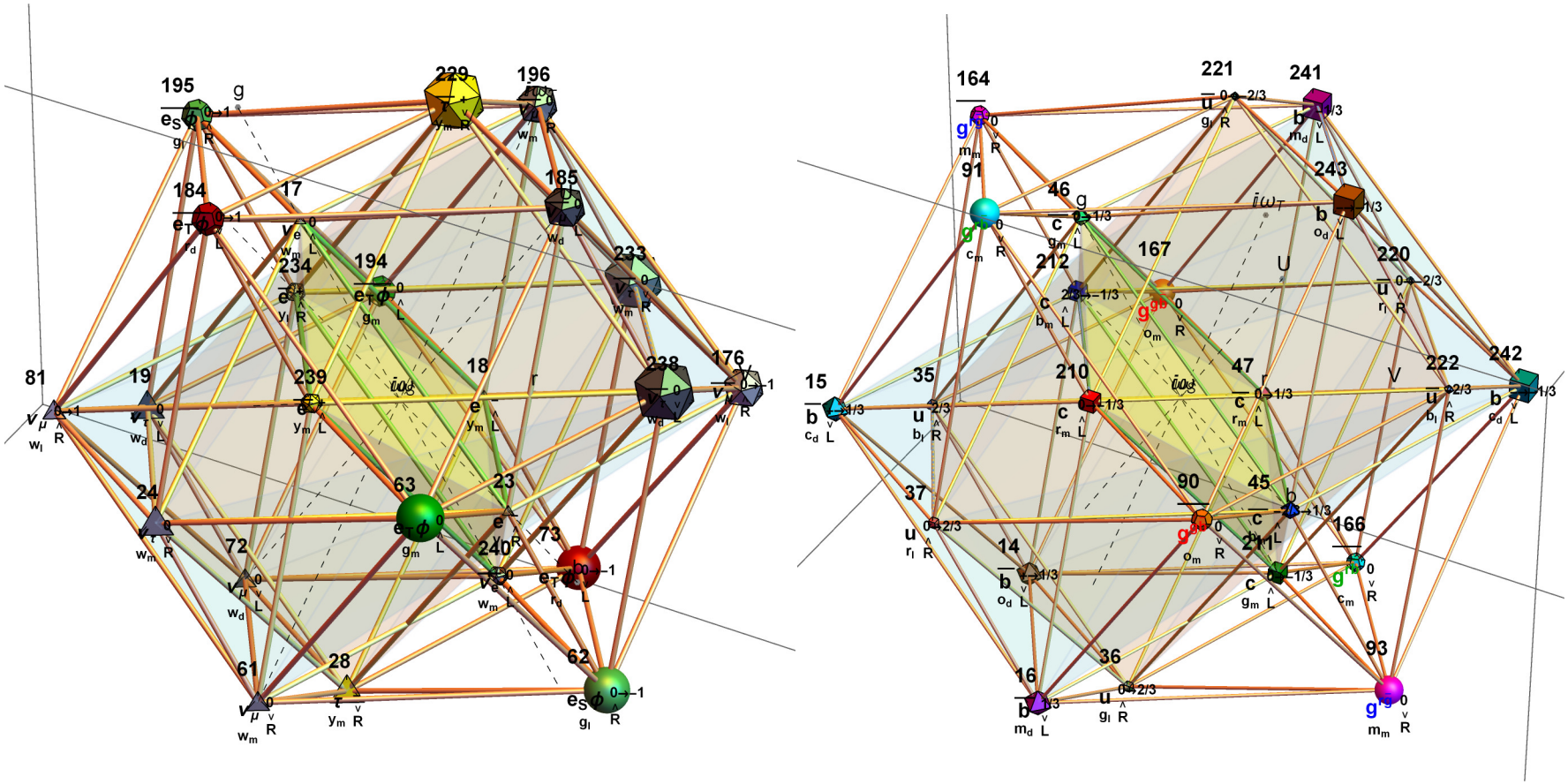
H4 (left) and H4Φ (right) Snub 24-Cell (1st of 4 rotations by $\frac{\pi}{5}$ from the 0th rotation shown above)



ColumnForm[MatrixForm[{Row@{Column[{#, pDisp@#, pLbl@#}, Center]} & /@ #}] & /@ {h4snub161, h4snub81, h4@snub161, h4@snub81}]

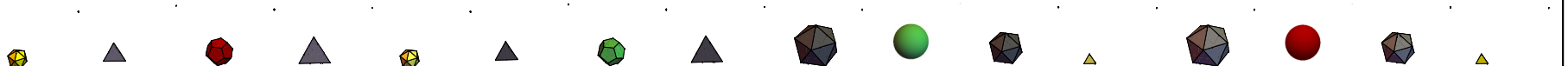

137 92 155 54 203 102 165 120
u g_m 0→2/3 ^ L
Φx_1 o_l 0→−2/3 ^ R
t b_m 2/3 v R
μ̄ y_m +→0 ^ L
μ y_m −→0 ^ L
t̄ b_m −2/3 v R
Φx_1 o_l 0→2/3 ^ R
ū g_m 0→−2/3 ^ L
232 124 174 59 99 12 172 57 200 85 245 158 198 83 133 25
b o_m −→−1/3 ^ L
d o_m −→−1/3 v R
Φx_2 c_m 0→1/3 ^ L
s̄ c_m +→1/3 ^ L
τ y_m − ^ L
e y_m − v R
Φx_3 m_d 0→−1/3 v L
c b_m 2/3→−1/3 v R
c̄ b_m −2/3→1/3 v R
Φx_3 m_d 0→1/3 v L
ē y_m + v R
τ̄ y_m + ^ L
s c_m −→−1/3 ^ L
Φx_2 c_m 0→−1/3 ^ L
d̄ o_m +→1/3 v R
b̄ o_m +→1/3 ^ L
49 34 71 21 236 186 223 208
e_Tφ g_d 0→−1 v L
d̄ m_d 1/3 v L
s̄ o_l +→1/3 ^ R
b̄ c_l +→1/3 ^ R
b c_l −→−1/3 ^ R
s o_l −→−1/3 ^ R
d m_d −1/3 v L
e_Tφ g_d 0→1 v L
39 64 142 42 160 106 75 126 131 182 151 97 215 115 193 218
e_Sφ r_m 0 ^ L
ω_L r_m 0→1 v R
d m_m −1/3 ^ L
s o_m −→−1/3 v R
t g_m 0→−1/3 ^ L
u b_d 2/3 v L
c r_l 0→2/3 ^ R
t g_l 0→2/3 ^ R
t̄ g_l 0→−2/3 ^ R
c̄ r_l 0→−2/3 ^ R
ū b_d −2/3 v L
t̄ g_m 0→1/3 ^ L
s̄ o_m +→1/3 v R
d̄ m_m 1/3 ^ L
ω_L r_m 0→−1 v R
e_Sφ r_m 0 ^ L

H4 (left) and H4Φ (right) Snub 24-Cell (2nd of 4 rotations by $\frac{\pi}{5}$)

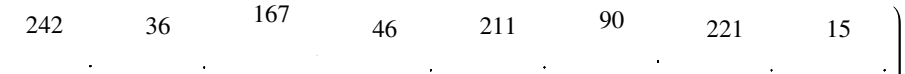


ColumnForm[MatrixForm[{Row@{Column[{#, pDisp@#, pLbl@#}, Center]} & /@ #}] & /@ {h4snub162, h4snub82, h4#snub162, h4#snub82}]

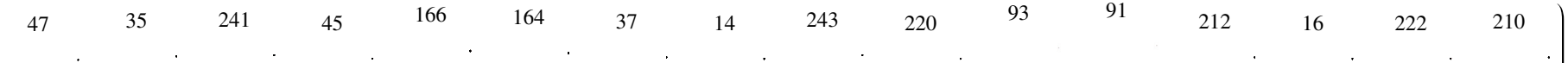
176	194	17	28	229	240	63	81
$\overline{\nu_\mu}^{\,0\to-1}_{w_l\ \hat{}\ R}$	$\overline{e_T\phi}^{\,0}_{g_m\ \hat{}\ L}$	$\nu_e^{\,0}_{w_m\ \hat{}\ L}$	$\tau^{\,-}_{y_m\ \check{}\ R}$	$\overline{\tau}^{\,+}_{y_m\ \check{}\ R}$	$\overline{\nu_e}^{\,0}_{w_m\ \hat{}\ L}$	$e_T\phi^{\,0}_{g_m\ \hat{}\ L}$	$\nu_\mu^{\,0\to1}_{w_l\ \hat{}\ R}$



239	61	184	24	234	72	195	19	238	62	185	23	233	73	196	18
$\overline{e}^{\,+}_{y_m\ \hat{}\ L}$	$\nu_\mu^{\,0}_{w_m\ \check{}\ R}$	$\overline{e_T\phi}^{\,0\to1}_{r_d\ \check{}\ L}$	$\nu_\tau^{\,0}_{w_m\ \check{}\ R}$	$\overline{e}^{\,+}_{y_l\ \hat{}\ R}$	$\nu_\mu^{\,0}_{w_d\ \check{}\ L}$	$\overline{e_S\phi}^{\,0\to1}_{g_l\ \hat{}\ R}$	$\nu_\tau^{\,0}_{w_d\ \check{}\ L}$	$\overline{\nu_\tau}^{\,0}_{w_d\ \check{}\ L}$	$e_S\phi^{\,0\to-1}_{g_l\ \hat{}\ R}$	$\overline{\nu_\mu}^{\,0}_{w_d\ \check{}\ L}$	$e^{\,-}_{y_l\ \hat{}\ R}$	$\overline{\nu_\tau}^{\,0}_{w_m\ \check{}\ R}$	$e_T\phi^{\,0\to-1}_{r_d\ \check{}\ L}$	$\overline{\nu_\mu}^{\,0}_{w_m\ \check{}\ R}$	$e^{\,-}_{y_m\ \hat{}\ L}$

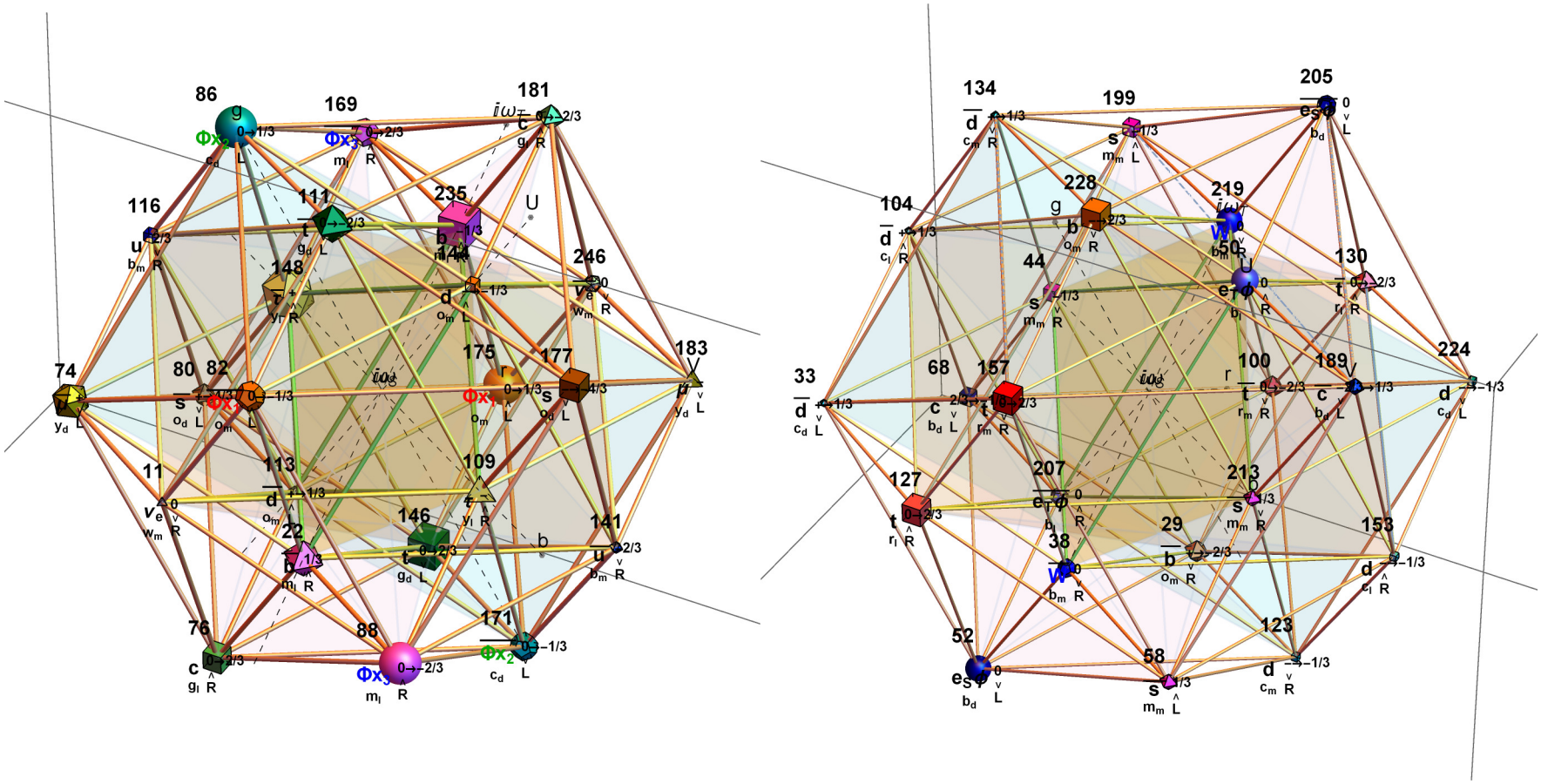


242	36	167	46	211	90	221	15
$b^{\,-\to-1/3}_{c_d\ \check{}\ L}$	$u^{\,0\to2/3}_{g_l\ \hat{}\ R}$	$g^{g\bar{b}\ 0}_{o_m\ \check{}\ R}$	$\overline{c}^{\,0\to1/3}_{g_m\ \hat{}\ L}$	$c^{\,0\to-1/3}_{g_m\ \hat{}\ L}$	$\overline{g^{g\bar{b}}}^{\,0}_{o_m\ \check{}\ R}$	$\overline{u}^{\,0\to-2/3}_{g_l\ \hat{}\ R}$	$\overline{b}^{\,+\to1/3}_{c_d\ \check{}\ L}$



47	35	241	45	166	164	37	14	243	220	93	91	212	16	222	210
$\overline{c}^{\,0\to1/3}_{r_m\ \hat{}\ L}$	$u^{\,2/3}_{b_l\ \hat{}\ R}$	$b^{\,-1/3}_{m_d\ \check{}\ L}$	$\overline{c}^{\,-2/3\to1/3}_{b_m\ \hat{}\ L}$	$\overline{g^{r\bar{b}}}^{\,0}_{c_m\ \check{}\ R}$	$\overline{g^{r\bar{g}}}^{\,0}_{m_m\ \check{}\ R}$	$u^{\,0\to2/3}_{r_l\ \hat{}\ R}$	$\overline{b}^{\,+\to1/3}_{o_d\ \check{}\ L}$	$b^{\,-\to-1/3}_{o_d\ \check{}\ L}$	$\overline{u}^{\,0\to-2/3}_{r_l\ \hat{}\ R}$	$g^{r\bar{g}\ 0}_{m_m\ \check{}\ R}$	$g^{r\bar{b}\ 0}_{c_m\ \check{}\ R}$	$c^{\,2/3\to-1/3}_{b_m\ \hat{}\ L}$	$\overline{b}^{\,1/3}_{m_d\ \check{}\ L}$	$\overline{u}^{\,-2/3}_{b_l\ \hat{}\ R}$	$c^{\,0\to-1/3}_{r_m\ \hat{}\ L}$

H4 (left) and H4Φ (right) Snub 24-Cell (3rd of 4 rotations by $\frac{\pi}{5}$)

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ColumnForm[MatrixForm[{Row@{Column[{#, pDisp@#, pLbl@#}, Center]} & /@ #}] & /@ {h4snub163, h4snub83, h4#snub163, h4#snub83}]
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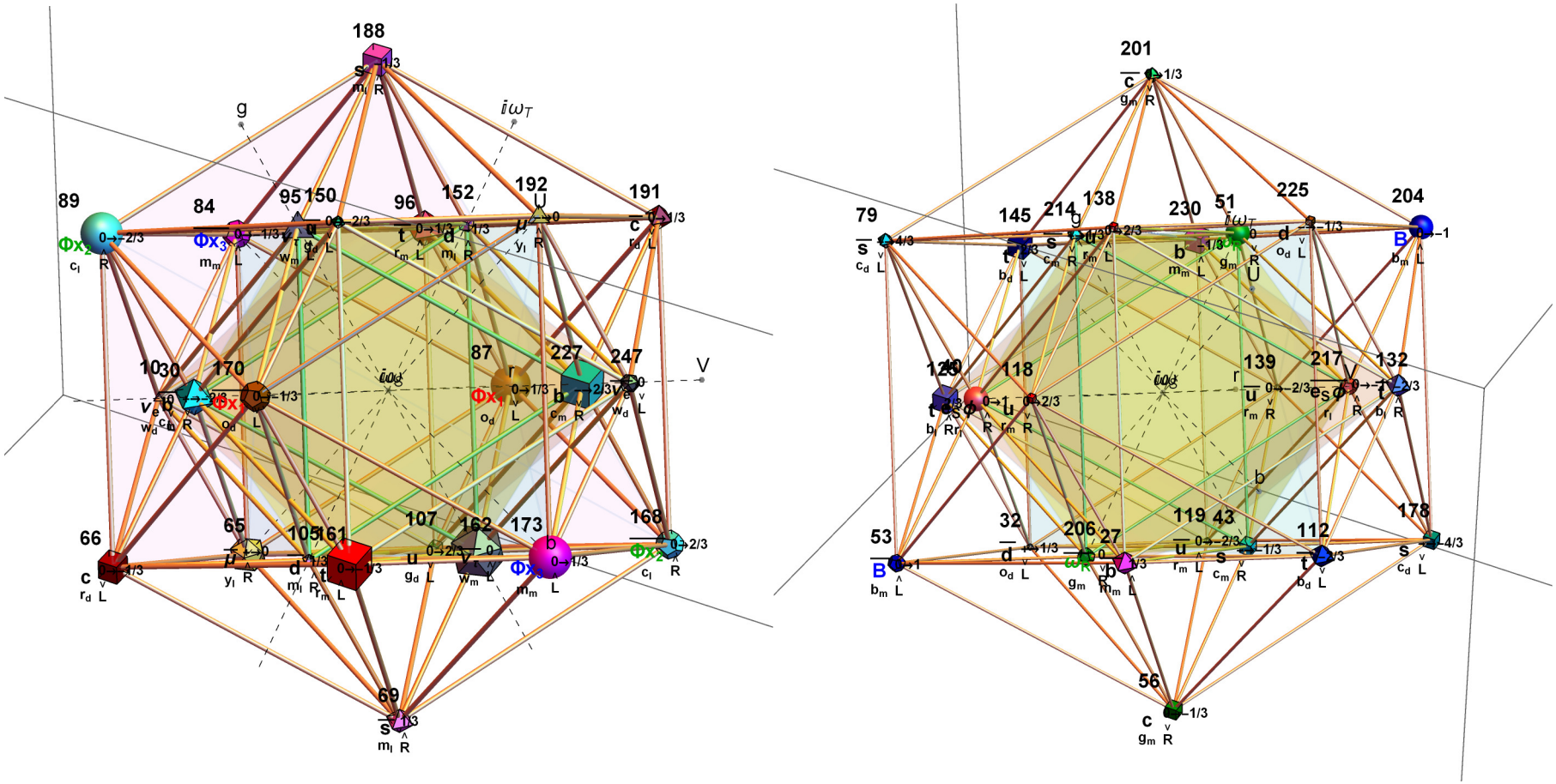
183	169	144	146	111	113	88	74
$\mu_{y_d}^{-\,\vee\,L}$	$\overline{\Phi x_3}_{m_l}^{0\to 2/3\,\wedge\,R}$	$d_{o_m}^{-\to -1/3\,\wedge\,L}$	$t_{g_d}^{0\to 2/3\,\vee\,L}$	$\overline{t}_{g_d}^{0\to -2/3\,\vee\,L}$	$\overline{d}_{o_m}^{+\to 1/3\,\wedge\,L}$	$\Phi x_3{}_{m_l}^{0\to -2/3\,\wedge\,R}$	$\overline{\mu}_{y_d}^{+\,\vee\,L}$

11	76	82	22	116	80	86	148	109	171	177	141	235	175	181	246
$\nu_{e_{w_m}}^{0\,\vee\,R}$	$c_{g_l}^{0\to 2/3\,\wedge\,R}$	$\overline{\Phi x_1}_{o_m}^{0\to -1/3\,\wedge\,L}$	$\overline{b}_{m_l}^{1/3\,\wedge\,R}$	$u_{b_m}^{2/3\,\vee\,R}$	$\overline{s}_{o_d}^{+\to 4/3\,\vee\,L}$	$\Phi x_2{}_{c_d}^{0\to 1/3\,\vee\,L}$	$\overline{\tau}_{y_l}^{+\,\wedge\,R}$	$\tau_{y_l}^{-\,\wedge\,R}$	$\overline{\Phi x_2}_{c_d}^{0\to -1/3\,\vee\,L}$	$s_{o_d}^{-\to -4/3\,\vee\,L}$	$\overline{u}_{b_m}^{-2/3\,\vee\,R}$	$b_{m_l}^{-1/3\,\wedge\,R}$	$\Phi x_1{}_{o_m}^{0\to 1/3\,\wedge\,L}$	$\overline{c}_{g_l}^{0\to -2/3\,\wedge\,R}$	$\overline{\nu_e}_{w_m}^{0\,\vee\,R}$

224	29	199	50	207	58	228	33
$d_{c_d}^{-\to -1/3\,\vee\,L}$	$\overline{b}_{o_m}^{+\to -2/3\,\vee\,R}$	$s_{m_m}^{-1/3\,\wedge\,L}$	$e_T\phi_{b_l}^{0\,\wedge\,R}$	$\overline{e_T\phi}_{b_l}^{0\,\wedge\,R}$	$\overline{s}_{m_m}^{1/3\,\wedge\,L}$	$b_{o_m}^{-\to 2/3\,\vee\,R}$	$\overline{d}_{c_d}^{+\to 1/3\,\vee\,L}$

130	44	205	153	100	134	38	68	189	219	123	157	104	52	213	127
$\overline{t}_{r_l}^{0\to -2/3\,\wedge\,R}$	$s_{m_m}^{-1/3\,\vee\,R}$	$\overline{e_S\phi}_{b_d}^{0\,\vee\,L}$	$d_{c_l}^{-\to -1/3\,\wedge\,R}$	$\overline{t}_{r_m}^{0\to -2/3\,\vee\,R}$	$\overline{d}_{c_m}^{+\to 1/3\,\vee\,R}$	$\overline{W}_{b_m}^{0\,\vee\,R}$	$c_{b_d}^{2/3\to -1/3\,\vee\,L}$	$\overline{c}_{b_d}^{-2/3\to 1/3\,\vee\,L}$	$W_{b_m}^{0\,\vee\,R}$	$d_{c_m}^{-\to -1/3\,\vee\,R}$	$t_{r_m}^{0\to 2/3\,\vee\,R}$	$\overline{d}_{c_l}^{+\to 1/3\,\wedge\,R}$	$e_S\phi_{b_d}^{0\,\vee\,L}$	$\overline{s}_{m_m}^{1/3\,\vee\,R}$	$t_{r_l}^{0\to 2/3\,\wedge\,R}$

H4 (left) and H4Φ (right) Snub 24-Cell (4th of 4 rotations by $\frac{\pi}{5}$)



ColumnForm[MatrixForm[{Row@{Column[{#, pDisp@#, pLbl@#}, Center]} & /@ #}] & /@ {h4snub164, h4snub84, h4#snub164, h4#snub84}]

227	188	247	87	170	10	69	30
							
$b^{-\to 2/3}_{c_m\ \check{R}}$	$s^{-1/3}_{m_l\ \hat{R}}$	$\overline{\nu_e}^{\,0}_{w_d\ \check{L}}$	$\Phi x_1{}^{0\to 1/3}_{o_d\ \check{L}}$	$\overline{\Phi x_1}^{\,0\to -1/3}_{o_d\ \check{L}}$	$\nu_e{}^{0}_{w_d\ \check{L}}$	$\overline{s}^{\,1/3}_{m_l\ \hat{R}}$	$\overline{b}^{+\to -2/3}_{c_m\ \check{R}}$

66	65	161	162	89	84	150	152	105	107	173	168	95	96	192	191
															
$c^{0\to -1/3}_{r_d\ \check{L}}$	$\overline{\mu}^{+\to 0}_{y_l\ \hat{R}}$	$t^{0\to -1/3}_{r_m\ \hat{L}}$	$\overline{\nu_\tau}^{\,0}_{w_m\ \hat{L}}$	$\Phi x_2{}^{0\to -2/3}_{c_l\ \hat{R}}$	$\overline{\Phi x_3}^{\,0\to -1/3}_{m_m\ \hat{L}}$	$\overline{u}^{\,0\to -2/3}_{g_d\ \check{L}}$	$d^{-1/3}_{m_l\ \hat{R}}$	$\overline{d}^{\,1/3}_{m_l\ \hat{R}}$	$u^{0\to 2/3}_{g_d\ \check{L}}$	$\Phi x_3{}^{0\to 1/3}_{m_m\ \hat{L}}$	$\overline{\Phi x_2}^{\,0\to 2/3}_{c_l\ \hat{R}}$	$\nu_\tau{}^{0}_{w_m\ \hat{L}}$	$\overline{t}^{\,0\to 1/3}_{r_m\ \hat{L}}$	$\mu^{-\to 0}_{y_l\ \hat{R}}$	$\overline{c}^{\,0\to 1/3}_{r_d\ \check{L}}$

217	139	201	132	125	56	118	40
							
$\overline{e_S\phi}^{\,0\to -1}_{r_l\ \hat{R}}$	$\overline{u}^{\,0\to -2/3}_{r_m\ \check{R}}$	$\overline{c}^{\,0\to 1/3}_{g_m\ \check{R}}$	$\overline{t}^{\,-2/3}_{b_l\ \hat{R}}$	$t^{2/3}_{b_l\ \hat{R}}$	$c^{0\to -1/3}_{g_m\ \check{R}}$	$u^{0\to 2/3}_{r_m\ \check{R}}$	$e_S\phi^{\,0\to 1}_{r_l\ \hat{R}}$

204	51	225	178	230	138	43	145	112	214	119	27	79	32	206	53
															
$B^{0\to -1}_{b_m\ \hat{L}}$	$\omega_R{}^{0}_{g_m\ \check{R}}$	$d^{-\to -1/3}_{o_d\ \check{L}}$	$s^{-\to -4/3}_{c_d\ \check{L}}$	$b^{-1/3}_{m_m\ \hat{L}}$	$u^{0\to 2/3}_{r_m\ \hat{L}}$	$s^{-\to -1/3}_{c_m\ \check{R}}$	$t^{2/3}_{b_d\ \check{L}}$	$\overline{t}^{\,-2/3}_{b_d\ \check{L}}$	$\overline{s}^{\,+\to 1/3}_{c_m\ \check{R}}$	$\overline{u}^{\,0\to -2/3}_{r_m\ \hat{L}}$	$\overline{b}^{\,1/3}_{m_m\ \hat{L}}$	$\overline{s}^{\,+\to 4/3}_{c_d\ \check{L}}$	$\overline{d}^{\,+\to 1/3}_{o_d\ \check{L}}$	$\overline{\omega_R}^{\,0}_{g_m\ \check{R}}$	$\overline{B}^{\,0\to 1}_{b_m\ \hat{L}}$