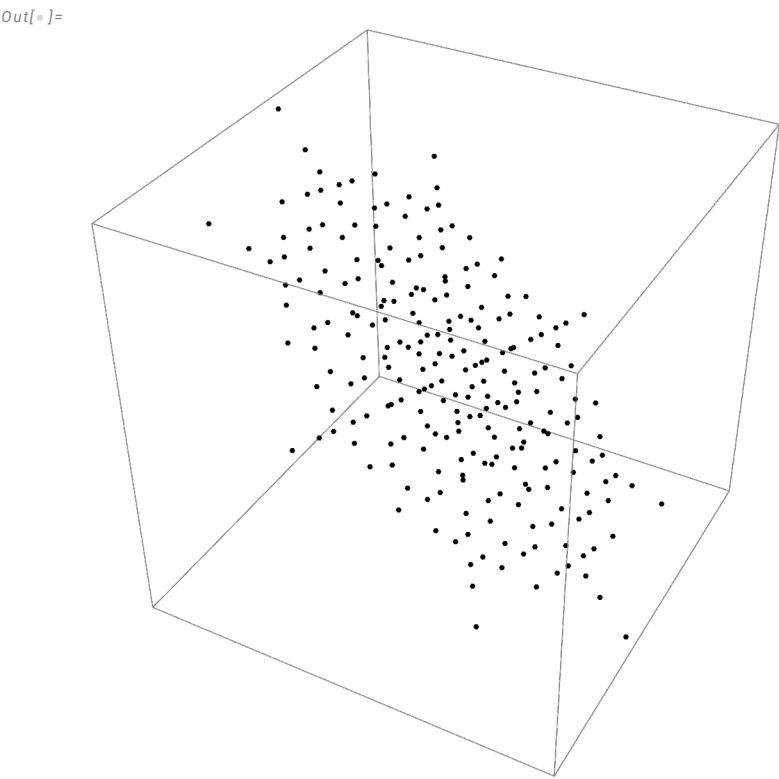
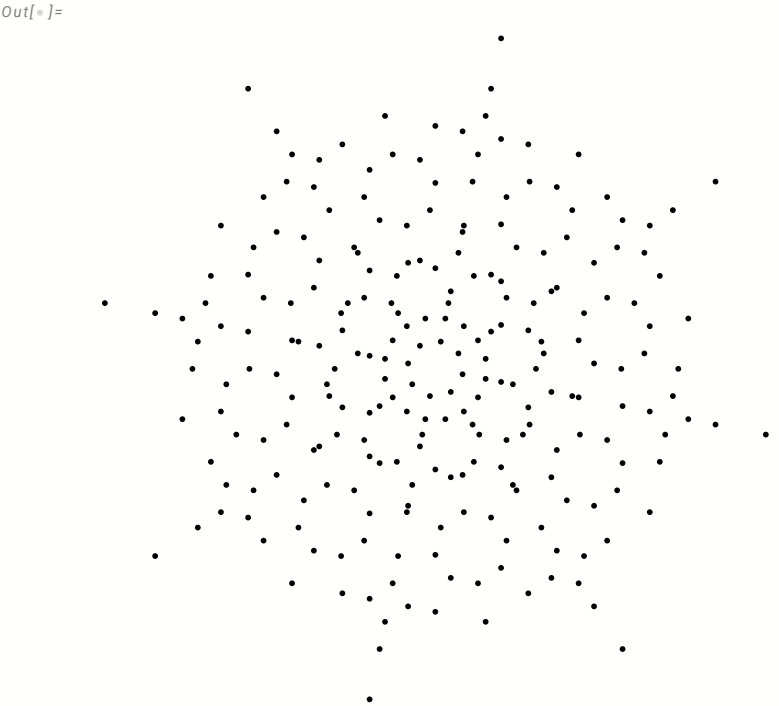



```
In[*]:= N@Log@ $\frac{27}{19}$ 
(* M2min=0 *)
60×4
projBWe8421 = Table[round[#.i], {i, (**)projBW8(*)H430LR8(**)E830LR8(**)}] & /@ (* e8421**)E8LatticeCoordinates[[1]] (**);
Length@%
Graphics@Point[projBWe8421[[All, ;; 2]]]
Graphics3D@Point[projBWe8421]
```

Out[*]=
0.351398

Out[*]=
240

Out[*]=
240



```
In[*]:= (* E8241 <x,x≥4 2160 *)
```

```
In[*]:= (* M2^min=0+M2^max=Log@16/7 *)
(60 + 480)
%*4
N@Log@27/19
projBWe8241 = Table[round[#,i], {i, (*projBW8**)H430LR8(*)E830LR8**)}] & /@ (*e8241**)E8LatticeCoordinates[[2]] (**);
Length@%
Graphics@Point[projBWe8241[[All, ;; 2]]]

Out[*]=
540

Out[*]=
2160

Out[*]=
0.351398

Out[*]=
2160

Out[*]=
.

: : : :

: : : : : : :
: : : : : : :
: : : : : : :
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: : : : : : :
: : : :

.
```

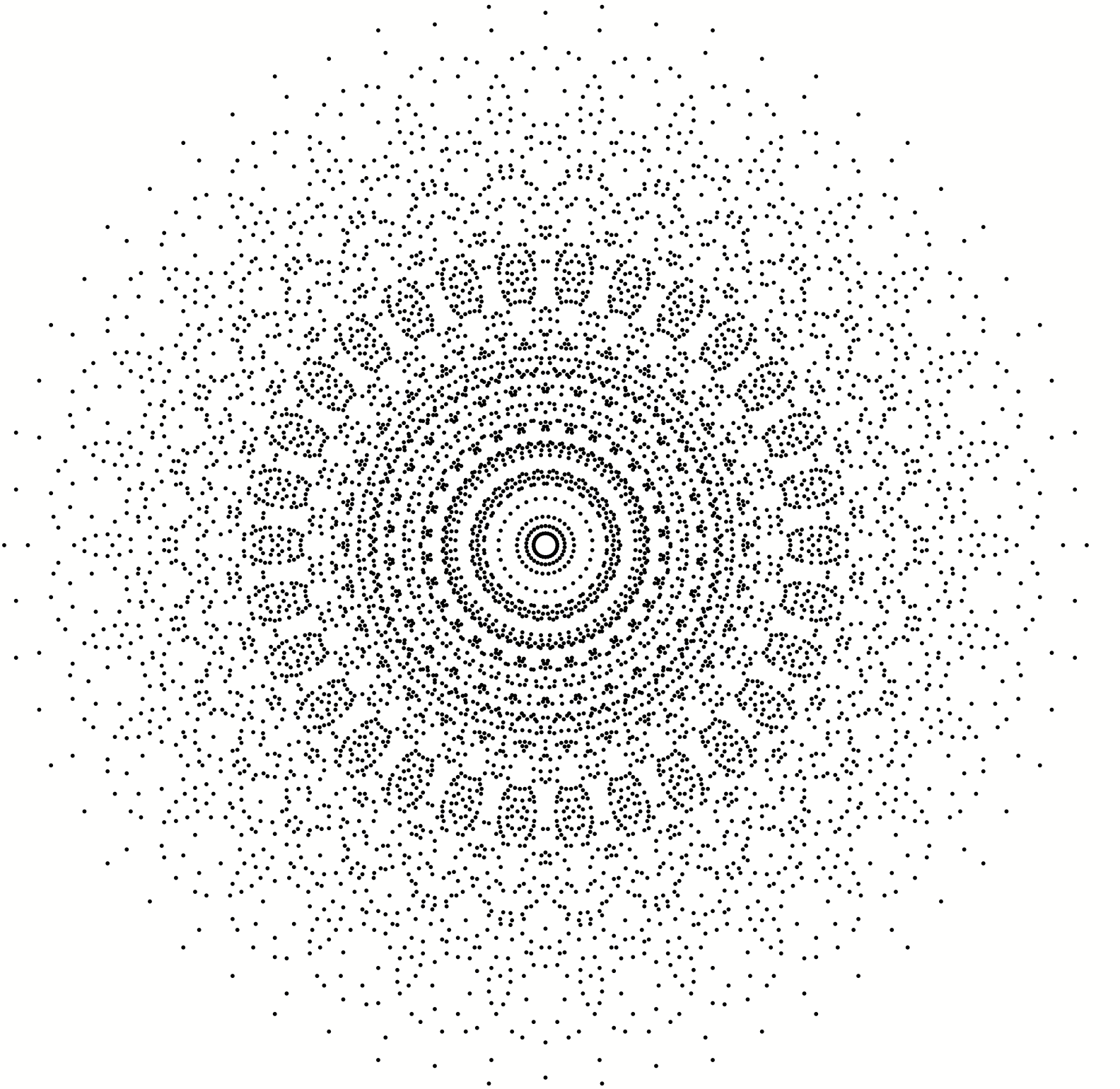
```
In[*]:= (* Rectified E8421 <x,x≥6 6720 *)
```

```
In[ ]:= (* Log@27/19+Log@9/5 *)  
      (960 + 720)  
      %×4  
projBwe8Rect = Table[round[#,i], {i, (*)projBW8(**)H430LR8(**)E830LR8(**)}] & /@ (*)e8Rect(**)E8LatticeCoordinates[[3]] (**);  
Length@%  
Graphics@Point[projBwe8Rect[[All, ;; 2]]]
```

Out[]=
1680

Out[]=
6720

Out[]=
6720



```
In[ ]:= (* E8142 <x,x>8 17280 *)
```

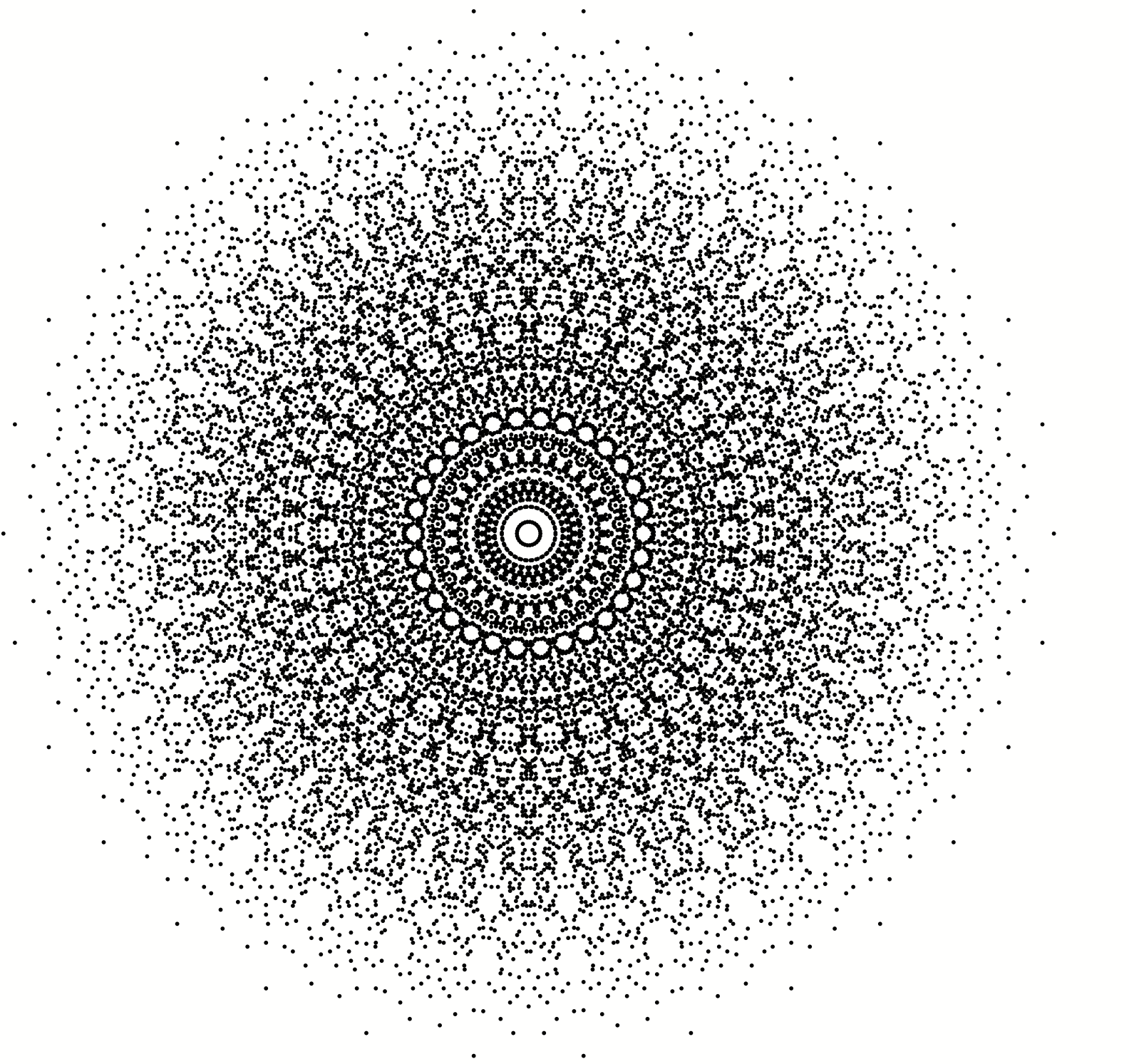
```
In[*]:= (* M2min=0 + Log@256/139 *)
(3840 + 480)
%×4
projBWe8142 = Table[round[#,i], {i, (*)projBW8(**)H430LR8(**)E830LR8(**)}] & /@ (*)e8142(**)E8LatticeCoordinates[[4]] (**);
Length@%
Graphics@Point[projBWe8142[[All, ;; 2]]]

Out[*]=
4320

Out[*]=
17280

Out[*]=
17520

Out[*]=
```



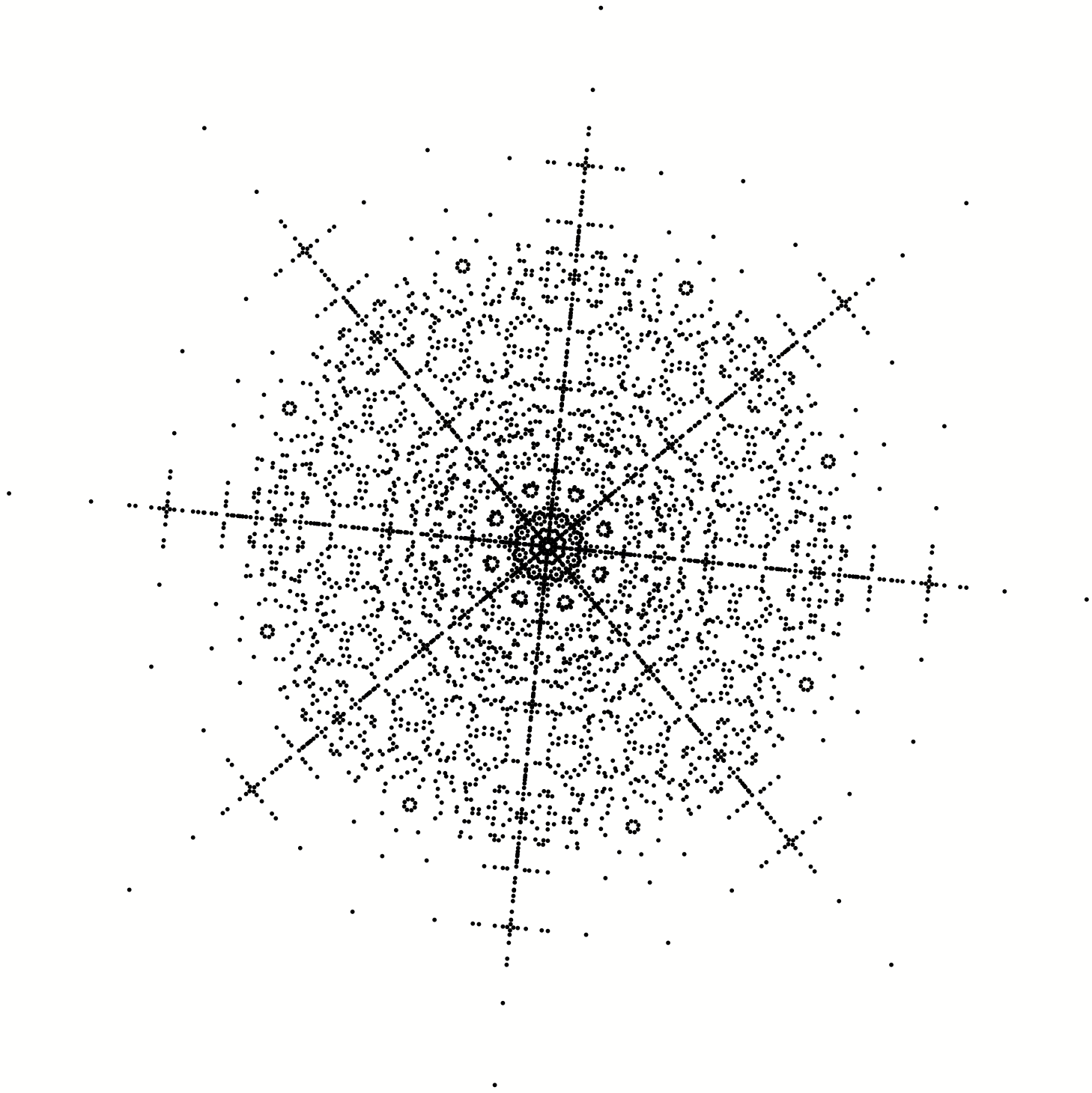
```
In[*]:= (* BW16 <→x,x≥4 4320 *)
```

```
In[*]:= (* M2^min=0+M2^max=Log@16/7 4320 *)
1080 × 4
(**)
projBW4320 = Table[round[#,i], {i, (**)projBW16a(*)H430LR16(**)E830LR16(**)}] & /@ Flatten[BWLatticeCoordinates1, 1];
Length@%
Graphics@Point[projBW4320[[All, ;; 2]]]
(*)
Graphics3D@Point[projBW4320]
```

Out[*]=
4320

Out[*]=
4320

Out[*]=



```
In[*]:= (* BW16 <vec{x},vec{x}>6 61440 *)
```

```
In[*]:= projBW16[[;; 2]]
```

Out[*]=

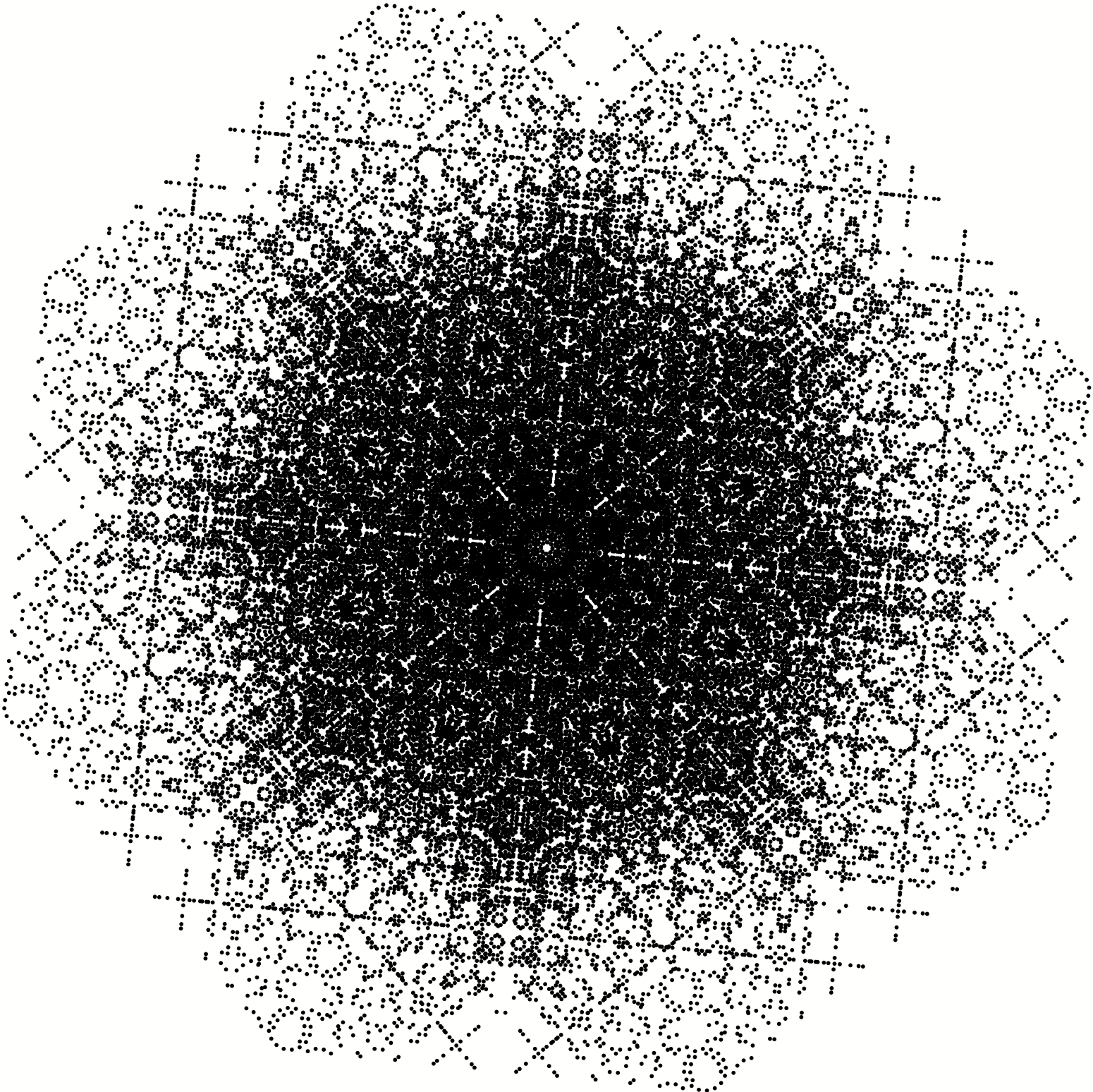
$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \cos(\frac{\pi}{8}) & \frac{1}{2\sqrt{2}} & \frac{1}{2} \sin(\frac{\pi}{8}) & 0 & -\frac{1}{2} \sin(\frac{\pi}{8}) & -\frac{1}{2\sqrt{2}} & -\frac{1}{2} \cos(\frac{\pi}{8}) & -\frac{1}{2} & -\frac{1}{2} \cos(\frac{\pi}{8}) & -\frac{1}{2\sqrt{2}} & -\frac{1}{2} \sin(\frac{\pi}{8}) & 0 & \frac{1}{2} \sin(\frac{\pi}{8}) & \frac{1}{2\sqrt{2}} & \frac{1}{2} \cos(\frac{\pi}{8}) \\ 0 & \frac{1}{2} \sin(\frac{\pi}{8}) & \frac{1}{2\sqrt{2}} & \frac{1}{2} \cos(\frac{\pi}{8}) & \frac{1}{2} & \frac{1}{2} \cos(\frac{\pi}{8}) & \frac{1}{2\sqrt{2}} & \frac{1}{2} \sin(\frac{\pi}{8}) & 0 & -\frac{1}{2} \sin(\frac{\pi}{8}) & -\frac{1}{2\sqrt{2}} & -\frac{1}{2} \cos(\frac{\pi}{8}) & -\frac{1}{2} & -\frac{1}{2} \cos(\frac{\pi}{8}) & -\frac{1}{2\sqrt{2}} & -\frac{1}{2} \sin(\frac{\pi}{8}) \end{pmatrix}$$

```
In[*]:= (* M2^max=log@9/2 *)
15 360 × 4
(**)
projBW61440 = Table[round[#,i], {i, (**) projBW16a(*) H430LR16(**) E830LR16(**) }} & /@ Flatten[BWLatticeCoordinates2, 1];
Length@%
Graphics@Point[projBW61440[[All, ;; 2]]]

Out[*]=
61 440

Out[*]=
61 440

Out[*]=
```



Set up Octonions

```
In[*]:= setFM@53;
triads

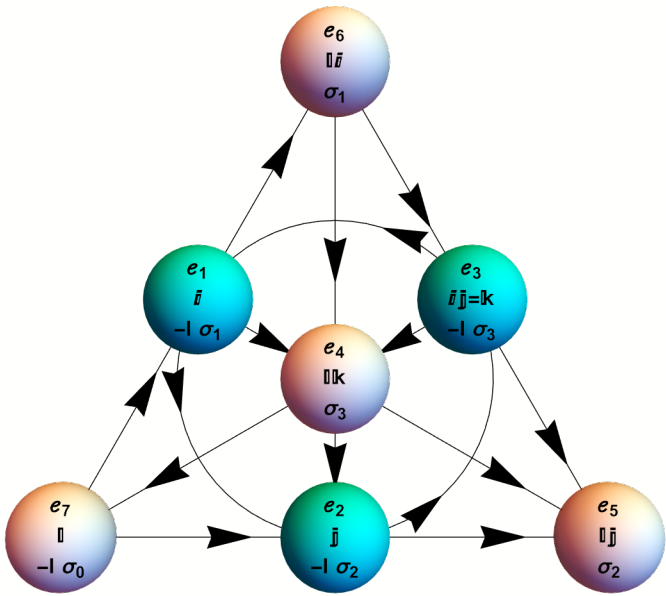
Out[*]=
( 1 2 3
  1 4 5
  1 6 7
  2 6 4
  2 5 7
  3 4 7
  3 5 6 )
```



```
In[*]:= fanoPlane
fmDispe
```

Out[*]=

Octonion Fano Plane in 3 forms: $\begin{pmatrix} 1 & e_{n \in \{1-7\}} \\ 2 & i j k \\ 3 & -I \sigma_{m \in \{1,2,3\}} \end{pmatrix}$



Out[*]//MatrixForm=

$$\begin{pmatrix} e_0 \mapsto 1 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ e_1 & -1 & e_3 & -e_2 & e_5 & -e_4 & e_7 & -e_6 \\ e_2 & -e_3 & -1 & e_1 & -e_6 & e_7 & e_4 & -e_5 \\ e_3 & e_2 & -e_1 & -1 & e_7 & e_6 & -e_5 & -e_4 \\ e_4 & -e_5 & e_6 & -e_7 & -1 & e_1 & -e_2 & e_3 \\ e_5 & e_4 & -e_7 & -e_6 & -e_1 & -1 & e_3 & e_2 \\ e_6 & -e_7 & -e_4 & e_5 & e_2 & -e_3 & -1 & e_1 \\ e_7 & e_6 & e_5 & e_4 & -e_3 & -e_2 & -e_1 & -1 \end{pmatrix}$$

```
In[*]:= kirmseClosedSete[[1]]
```

Out[*]=

$$\begin{pmatrix} \{e_1, -1, e_2, e_3\} & \{e_7, e_6, e_5, e_4\} \\ \{e_1, -1, e_4, e_5\} & \{e_7, e_6, e_2, e_3\} \\ \{e_1, -1, e_6, e_7\} & \{e_5, e_4, e_2, e_3\} \\ \{e_1, e_2, e_6, e_4\} & \{e_7, -1, e_5, e_3\} \\ \{e_1, e_2, e_5, e_7\} & \{e_6, -1, e_4, e_3\} \\ \{e_1, e_3, e_4, e_7\} & \{e_6, -1, e_2, e_5\} \\ \{e_1, e_3, e_5, e_6\} & \{e_7, -1, e_2, e_4\} \end{pmatrix}$$

Define Octonion based E8

```
In[*]:= octN
```

Out[*]=

$$\{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$$

```
In[*]:= Os = -Total[octN] * / 2
```

Out[*]=

$$\frac{1}{2} (-e_0 + e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7)$$

```
In[*]:= Os*
```

Out[*]=

$$-\frac{e_0}{2} - \frac{e_1}{2} - \frac{e_2}{2} - \frac{e_3}{2} - \frac{e_4}{2} - \frac{e_5}{2} - \frac{e_6}{2} - \frac{e_7}{2}$$

```
λ0240 = Flatten[octonion@# & /@ e8List[[e8]]];
```

```
(* Open this cell for the data *)
```

```
% // MatrixForm
```

Build of Kirmse Closed E8 from kirmseClosedSete

```
In[*]:= (* Pick one  $e_{n7=1-7}$  of 7 possible Kirmse rotations (aka. Maximal orders) and get
16  $D_4 = 4$  2D sign perms  $\times$  4 l/r 2D Pauli  $\sigma$  triads[[1]] U(1) position perms *)
octE816Closed1 =  $\sqrt{2}$  Sort@Flatten@Table[{
  (* 4 l/r 2D Pauli  $\sigma$  triads[[1]] U(1) position perms *)
  Total[# kirmseClosedSet $e$ [[1, lRQuatIndex, i, ;; 2]]],
  Total[# kirmseClosedSet $e$ [[1, lRQuatIndex, i, 3 ;;]]} & /@
  (* 4 2D sign perms *)
  dn[2] [[All, ;; 2],
  (* lQuat/rQuat on triads[[1]] *)
  {i, 2}] /. slRep;
% // MatrixForm

Out[*] // MatrixForm =

$$\begin{pmatrix} \sqrt{2} (-e_1 - 1) \\ \sqrt{2} (1 - e_1) \\ \sqrt{2} (e_1 - 1) \\ \sqrt{2} (e_1 + 1) \\ \sqrt{2} (-e_2 - e_3) \\ \sqrt{2} (e_2 - e_3) \\ \sqrt{2} (e_3 - e_2) \\ \sqrt{2} (e_2 + e_3) \\ \sqrt{2} (-e_4 - e_5) \\ \sqrt{2} (e_4 - e_5) \\ \sqrt{2} (e_5 - e_4) \\ \sqrt{2} (e_4 + e_5) \\ \sqrt{2} (-e_6 - e_7) \\ \sqrt{2} (e_6 - e_7) \\ \sqrt{2} (e_7 - e_6) \\ \sqrt{2} (e_6 + e_7) \end{pmatrix}$$


(* 224 = 16 4D sign perms  $\times$  14 l/r 4D Dirac  $\gamma$  quaternion SU(2) position perms *)
octE8224Closed1 = Flatten@Table[
  (* 16 sign perms *)
  Total[i #] & /@ pm@4 /. slRep,
  (* on 14=7 triad lQuat/rQuat position pairs *)
  {i, Flatten[kirmseClosedSet $e$ [[lRQuatIndex]], 1] }];

(* Open this cell for the data *)
% // MatrixForm

In[*]:= kirmseClosedE8Set1 = {octE816Closed1, octE8224Closed1};
Length /@ kirmseClosedE8Set1
kirmseClosedE81 = Join @@ kirmseClosedE8Set1;
Length@%

Out[*]=
{16, 224}

Out[*]=
240

In[*]:= kirmseClosedE81N = oct2List /@ Join[kirmseClosedE8Set1[[1]] /  $\sqrt{2}$ , kirmseClosedE8Set1[[2]]];

In[*]:= (* Kirmse Closed E8 *)

In[*]:= octE816Closed2 = Flatten@Table[pm i, {i, octN}, {pm, {1, -1}}];
octE8224Closed2 = octE8224Closed1 / 2;
kirmseClosedE8Set2 = {octE816Closed2, octE8224Closed2};
Length /@ kirmseClosedE8Set2
e8p = kirmseClosedE82 = Join @@ kirmseClosedE8Set2;
Length@%

Out[*]=
{16, 224}

Out[*]=
240
```

Define Octonion 24D Leech lattice vertices using Journal of Algebra 322 (2009) 2186–2190:Octonions and the Leech lattice by Robert A. Wilson

Applying the above symmetries it follows at once that Λ contains the following 196560 vectors of norm 4, where λ is a root of L and $j, k \in J = \{\pm i_t \mid t \in PL(7)\}$, and all permutations of the three coordinates are allowed:

$(2\lambda, 0, 0)$	Number:	3×240	=	720,	;
$(\lambda\bar{s}, \pm(\lambda\bar{s})j, 0)$	Number:	$3 \times 240 \times 16$	=	11520,	
$((\lambda s)j, \pm\lambda k, \pm(\lambda j)k)$	Number:	$3 \times 240 \times 16 \times 16$	=	184320.	

```
In[*]:= 3  $\times$  240 (1 + 16 + 162)
Out[*]=
196560
```



```
In[*]:= Δ0720 = Flatten[simplify /@ (2 {{#, 0 #, 0 #}, {0 #, #, 0 #}, {0 #, 0 #, #}}) & /@ λ0240, 1];
Length@%
%%[[;; 10]]
```

Out[*]=
720

Out[*]=

$$\begin{pmatrix} -e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1 & 0 & 0 \\ 0 & -e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1 & 0 \\ 0 & 0 & -e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1 \\ e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 + 1 & 0 & 0 \\ 0 & e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 + 1 & 0 \\ 0 & 0 & e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 + 1 \\ -e_1 + e_2 - e_3 - e_4 - e_5 - e_6 - e_7 + 1 & 0 & 0 \\ 0 & -e_1 + e_2 - e_3 - e_4 - e_5 - e_6 - e_7 + 1 & 0 \\ 0 & 0 & -e_1 + e_2 - e_3 - e_4 - e_5 - e_6 - e_7 + 1 \\ -e_1 - e_2 + e_3 - e_4 - e_5 - e_6 - e_7 + 1 & 0 & 0 \end{pmatrix}$$

```
In[*]:= Δ011520 = Flatten[Table[simplify /@ ({#◦0s*, (pm#◦0s*)◦j, 0 #}, {0 #, #◦0s*, (pm#◦0s*)◦j}, {(pm#◦0s*)◦j, 0 #, #◦0s*}}),
{j, octN}, {pm, {1, -1}}] & /@ λ0240, 3];
Length@%
%%[[;; 10]]
```

Out[*]=
11 520

Out[*]=

$$\begin{pmatrix} \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) & 0 \\ 0 & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) \\ \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) & 0 & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) \\ \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) & \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 + 3) & 0 \\ 0 & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) & \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 + 3) \\ \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 + 3) & 0 & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) \\ \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) & \frac{1}{2} (-3 e_1 + e_2 - e_3 + e_4 - e_5 + e_6 - e_7 - 1) & 0 \\ 0 & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) & \frac{1}{2} (-3 e_1 + e_2 - e_3 + e_4 - e_5 + e_6 - e_7 - 1) \\ \frac{1}{2} (-3 e_1 + e_2 - e_3 + e_4 - e_5 + e_6 - e_7 - 1) & 0 & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) \\ \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 - 3) & \frac{1}{2} (3 e_1 - e_2 + e_3 - e_4 + e_5 - e_6 + e_7 + 1) & 0 \end{pmatrix}$$

```
In[*]:= Δ0184320 = Flatten[Table[simplify /@
({{(#◦0s)◦j, pm1#◦k, pm2 (#◦j)◦k}, {pm2 (#◦j)◦k, (#◦0s)◦j, pm1#◦k}, {pm1#◦k, pm2 (#◦j)◦k, (#◦0s)◦j}}),
{j, octN}, {k, octN}, {pm1, {1, -1}}, {pm2, {1, -1}}] & /@ λ0240, 5];
Length@%
%%[[;; 10]]
```

Out[*]=
184 320

Out[*]=

$$\begin{pmatrix} 2 & \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) \\ \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & 2 & \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) \\ \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & 2 \\ 2 & \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + 1) \\ \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + 1) & 2 & \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) \\ \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + 1) & 2 \\ 2 & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + 1) & \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) \\ \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & 2 & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + 1) \\ \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + 1) & \frac{1}{2} (-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & 2 \\ 2 & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + 1) & \frac{1}{2} (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + 1) \end{pmatrix}$$

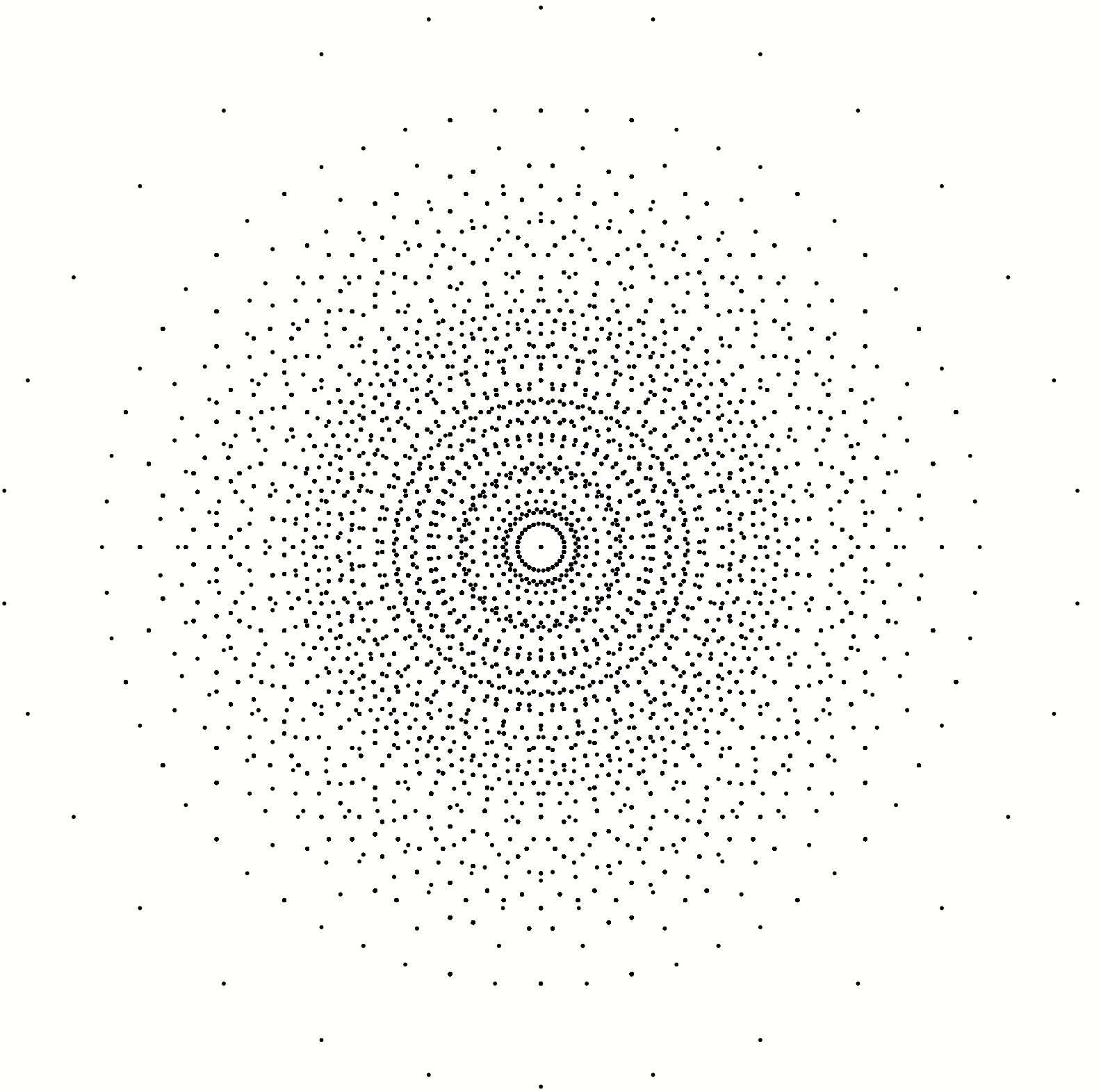
```
In[*]:= Δ0 = Join[Δ0720, Δ011520, Δ0184320];
Length@%
```

Out[*]=
196 560

```
(* Open this cell for the data *)
Δ24D = Flatten[oct2List@# & /@ #] & /@ Δ0;

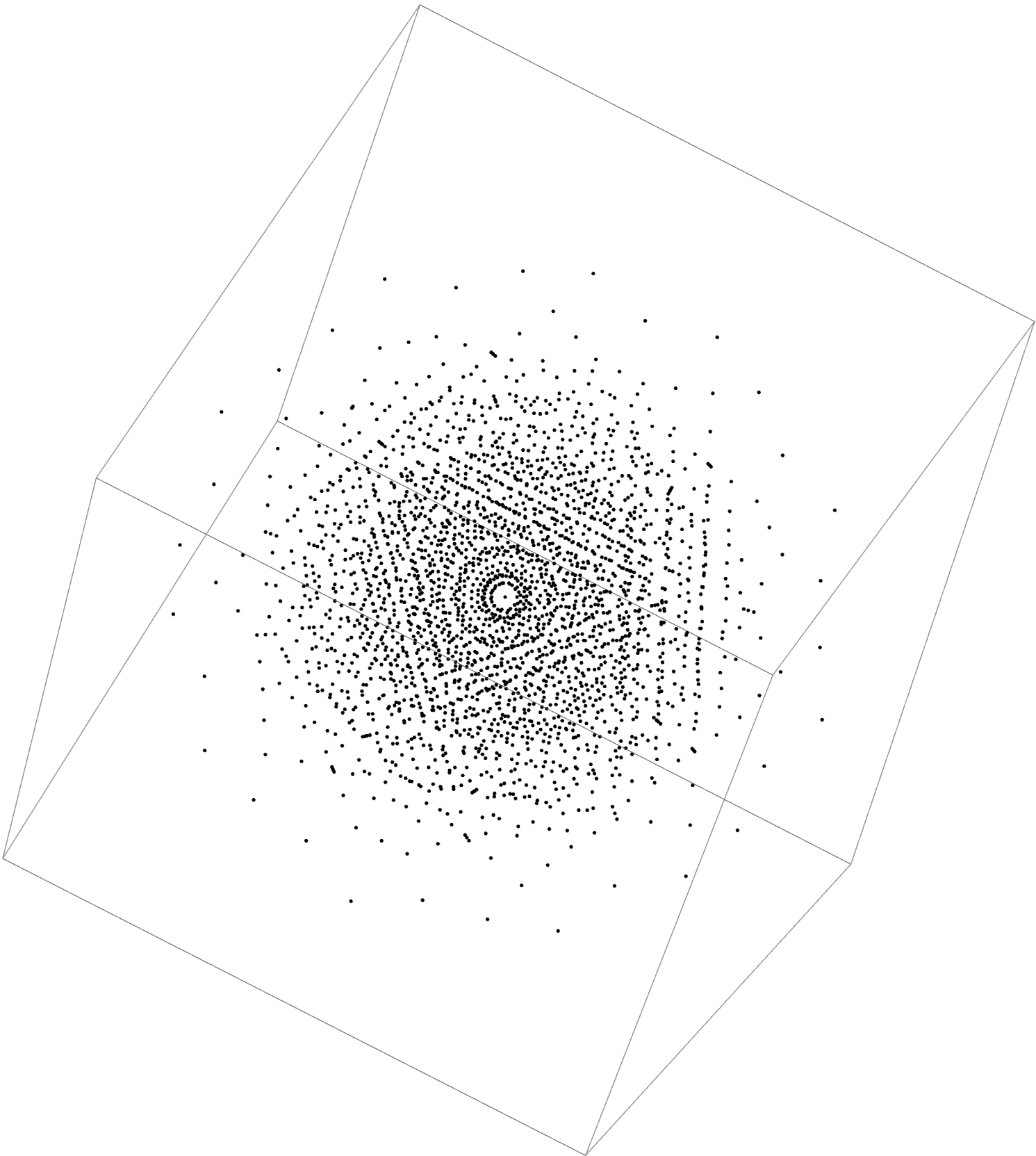
projΔ24D00 = Table[round[#, i], {i, (*) projBW24**} E830LR2400 (*) H430LR24**}] & /@ Δ24D;
Graphics@Point[projΔ24D00[[All, ;; 2]]]
```

Out[]=

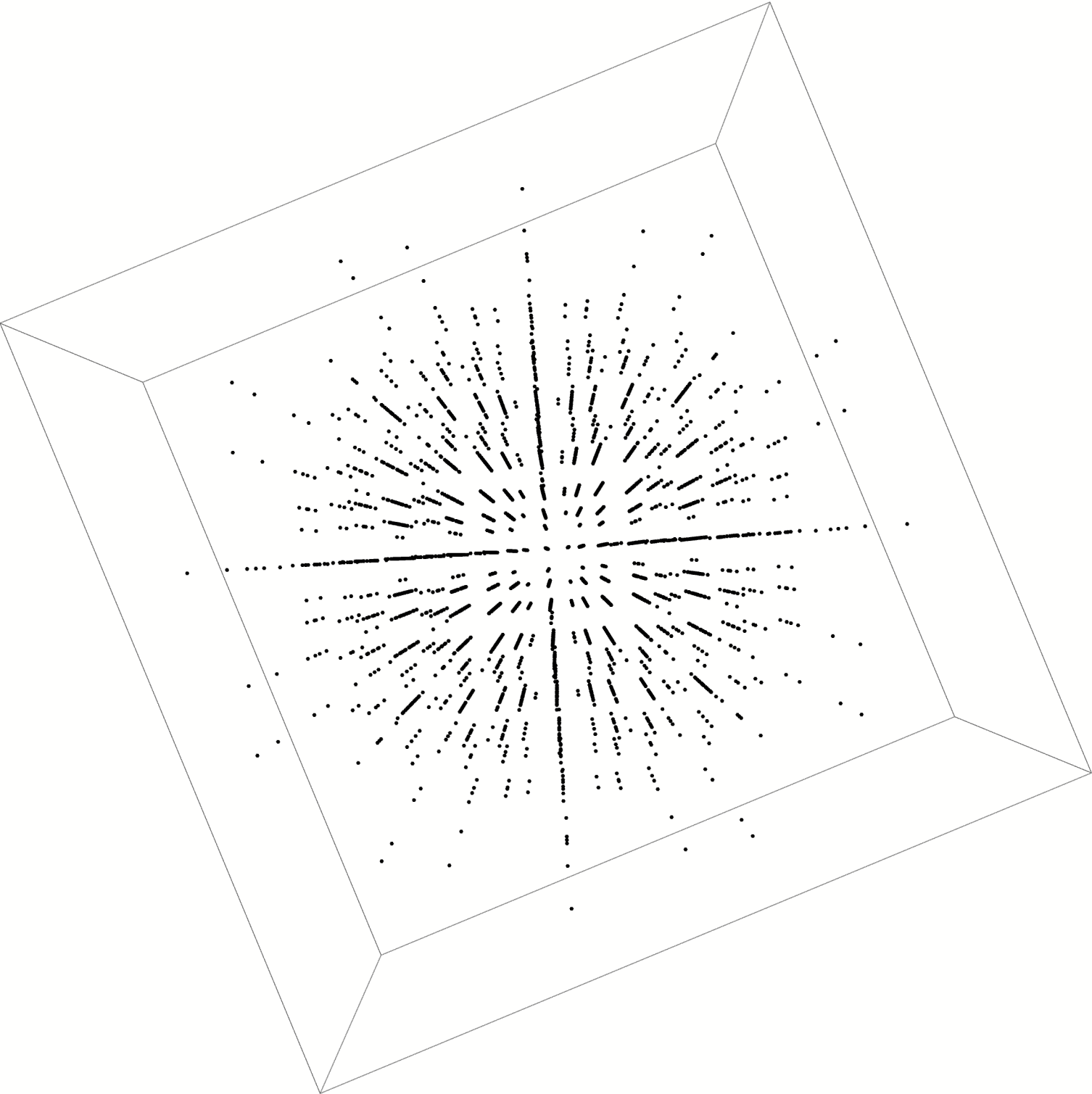


In[*]:= Graphics3D@Point[projΛ24D00]

Out[*]=



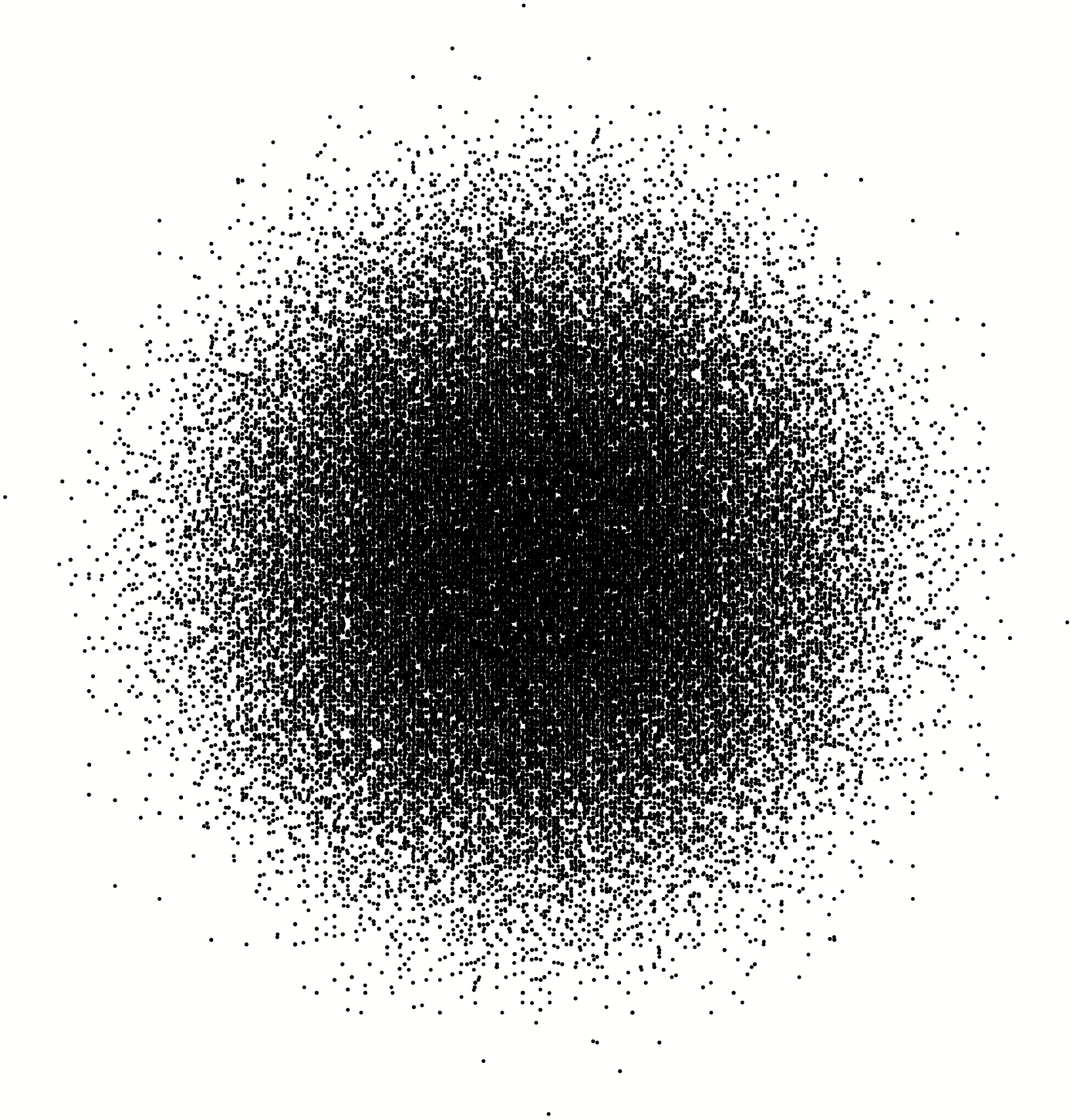
(* Same as above, only rotated to show the ortho8 H4 basis using the 3rd Z basis vector *)



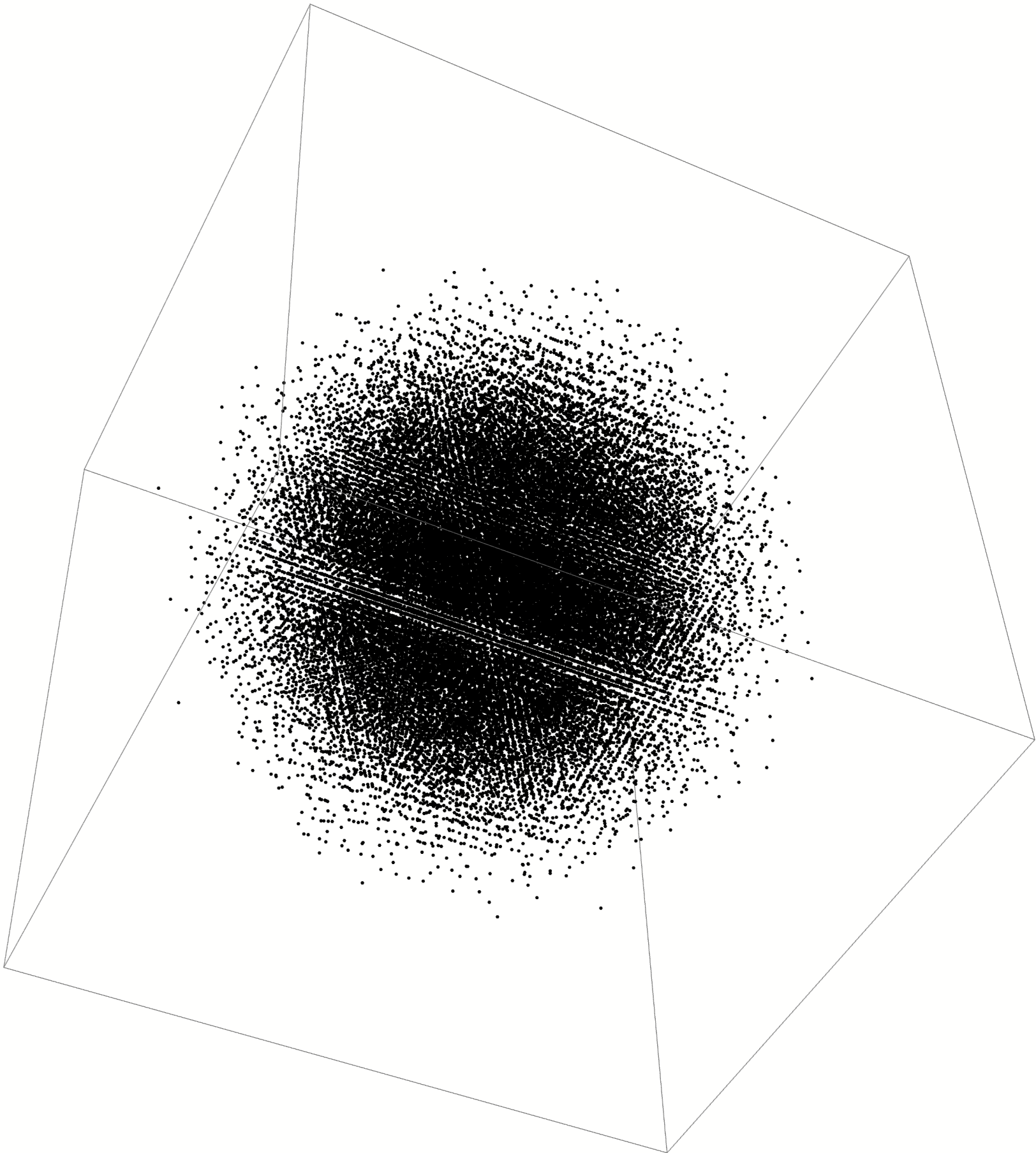
```

In[*]:= projΛ24D = Table[round[#,i], {i, (*projBW24**) E830LR24 (*H430LR24**) }] & /@ Λ24D;
Graphics@Point[projΛ24D[[All, ;; 2]]]
Graphics3D@Point[projΛ24D]
```

Out[]=

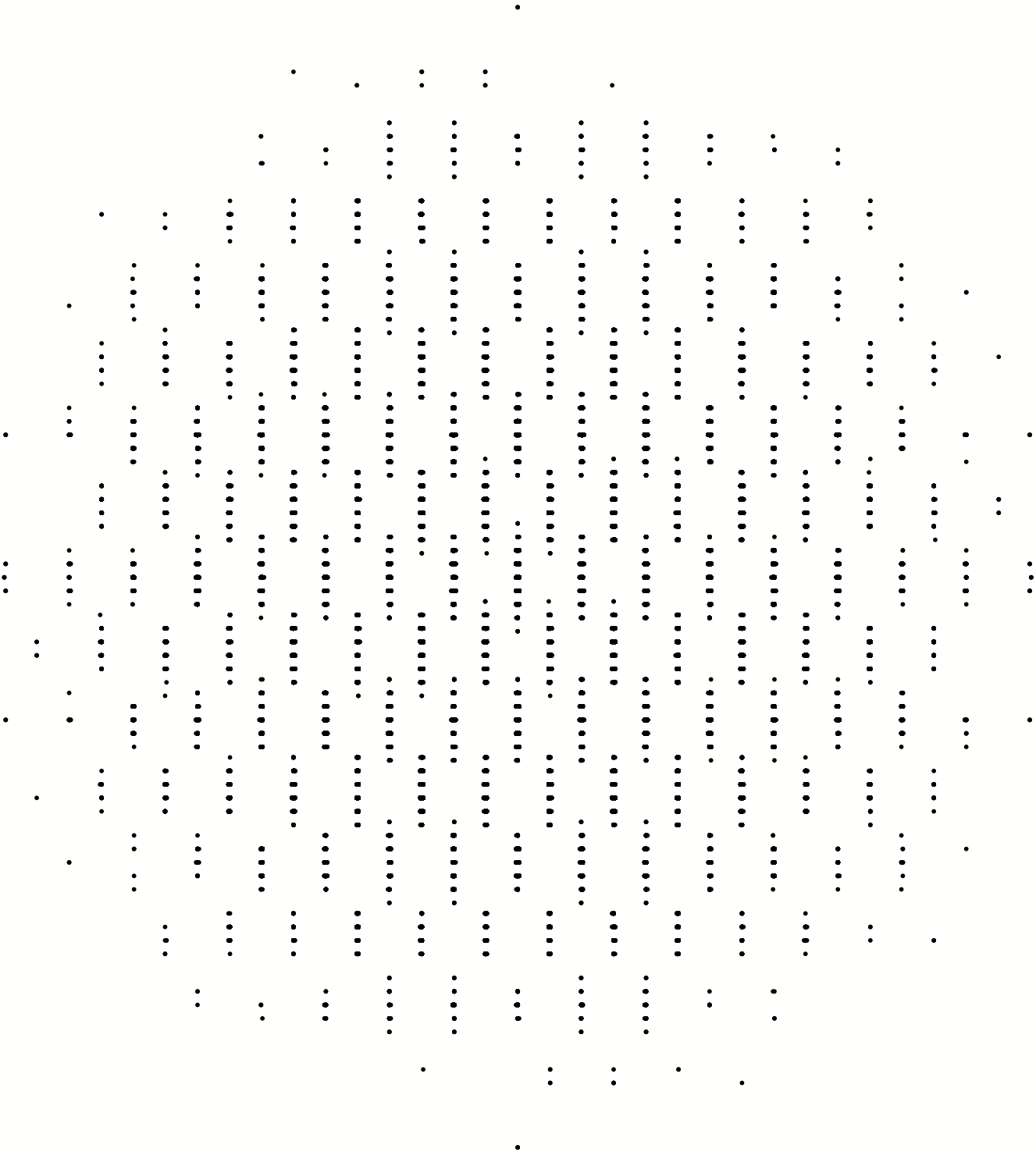


Out[]=

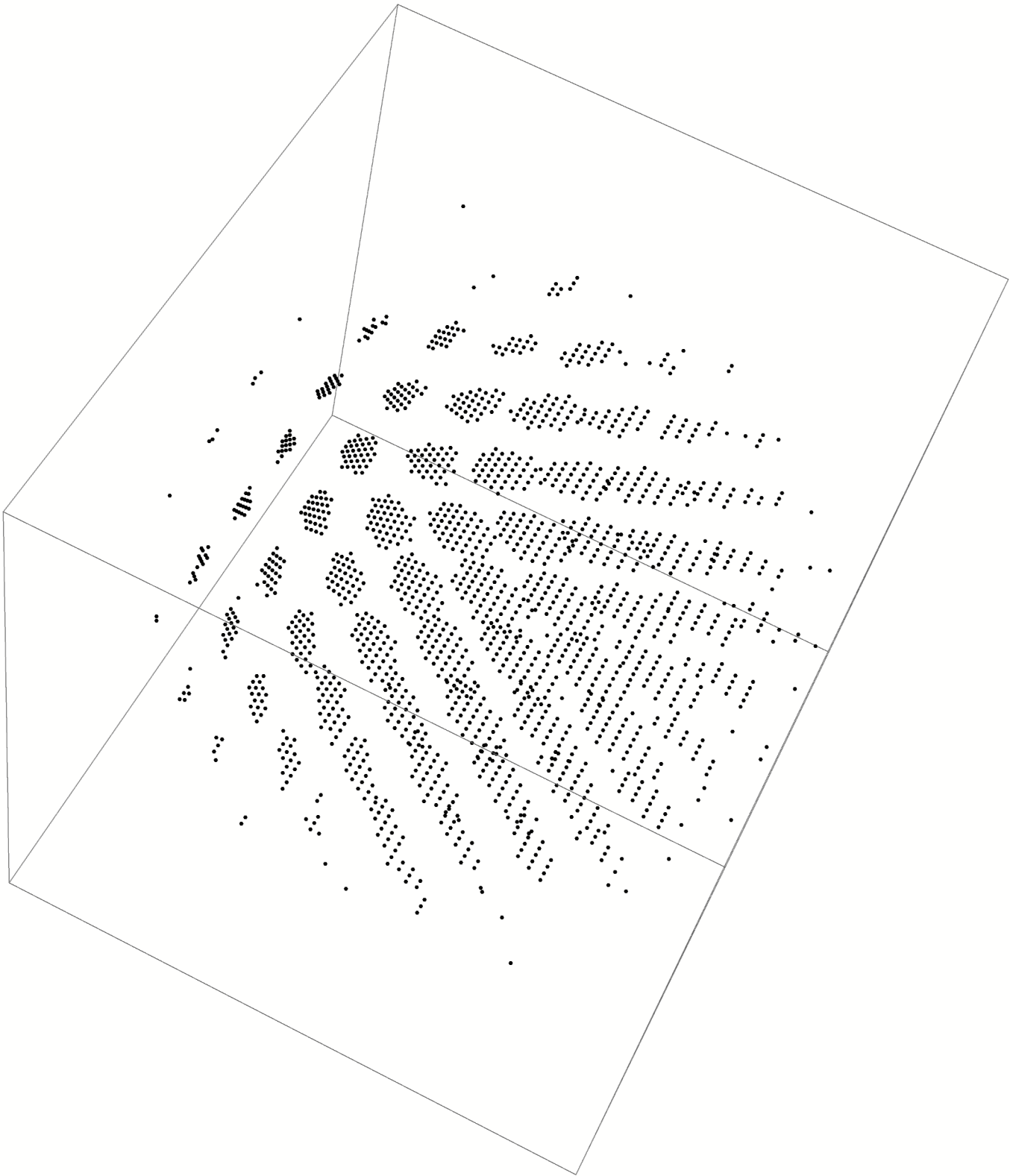


```
projΛ24D = Table[round[#,i], {i, (*)projBW24(**)E830LR24**)H430LR24(**)}] & /@ Λ24D;  
Graphics@Point[projΛ24D[[All, ;; 2]]  
Graphics3D@Point[projΛ24D]
```


Out[]=



Out[]:=



Define Octonion 16D Barnes-Wall lattice vertices using arXiv:hep-th/9501007v1: E8 Lattice to Λ_{16} by Geoffrey Dixon

Define

$$E_8^+ = \{\pm e_a\} \cup \{(\pm e_a \pm e_b \pm e_c \pm e_d)/2 : a, b, c, d \text{ distinct, } e_a(e_b(e_c e_d)) = \pm 1\},$$
$$a, b, c, d \in \{0, \dots, 7\}.$$

(11)

These $16 + 14 \times 16 = 240$ elements of the unit octonion 7-sphere form the inner shell of an E_8 lattice, which, like D_4^+ is closed under multiplication [5].

Define

$$E_8^- = \left\{ \frac{1}{2}(\pm e_a \pm e_b) : a, b \text{ distinct} \right\} \cup \left\{ \frac{1}{8}(\sum_{a=0}^7 \pm e_a) : \text{odd number of + 's} \right\},$$

(12)

These $112 + 128 = 240$ elements of the octonion 7-sphere of radius $1/\sqrt{2}$ also form the inner shell of an E_8 lattice, which, like D_4^- is not closed under multiplication. (The effect of my choice of octonion multiplication in (10) is in the definition of E_8^- ; there are choices that would require "odd number of +'s" to be changed to "even number of +'s" in (12); this would not change the order of that set, which would still be 128.)

From these two sets we can construct the inner shell of the 16-dimensional Λ_{16} lattice. In particular,

$$\begin{aligned} & \{ \langle U, 0 \rangle, \langle 0, V \rangle : U, V \in E_8^+ \} && (2 \times 240 = 480 \text{ elements}) \\ \cup & \{ \langle U, V \rangle : U, V \in E_8^-, UV^\dagger = \pm e_a \} && (16 \times 240 = 3840 \text{ elements}) \end{aligned}$$

(13)

```
In[ ]:= e8m = simplify[λ0240 / 2];

In[ ]:= e8pa = octE816Closed2 ∪ Flatten[perms[#, "None"] & /@
      Select[Subsets[octN, {4}], #[[1]] ∘ (#[[2]] ∘ (#[[3]] ∘ #[[4]])) == 1 || #[[1]] ∘ (#[[2]] ∘ (#[[3]] ∘ #[[4]])) == -1 &], 1] / 2;
Length@%

Out[ ]:=
240
```

```
In[*]:= BW0480a = Flatten[{{#, 0 #}, {0 #, #}} & /@ e8p, 1];
Length@%
BW0480a[[;; 10]]

Out[*]=
480

Out[*]=

$$\begin{pmatrix} e_0 & 0 \\ 0 & e_0 \\ -e_0 & 0 \\ 0 & -e_0 \\ e_1 & 0 \\ 0 & e_1 \\ -e_1 & 0 \\ 0 & -e_1 \\ e_2 & 0 \\ 0 & e_2 \end{pmatrix}$$


In[*]:= BW03840a = Select[Flatten[Table[{i, j}, {i, e8m}, {j, e8m}], 1],
MemberQ[ $\frac{\text{octE816Closed2}}{2}$  /. e0 -> 1, -#[[1]] o #[[2]]*] &];
Length@%
BW03840a[[;; 10]]

Out[*]=
3840

Out[*]=

$$\begin{pmatrix} \frac{1}{4}(-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{4}(-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) \\ \frac{1}{4}(-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{4}(e_1 + e_2 + e_3 - e_4 - e_5 - e_6 - e_7 + 1) \\ \frac{1}{4}(-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{4}(e_1 - e_2 - e_3 + e_4 + e_5 - e_6 - e_7 + 1) \\ \frac{1}{4}(-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{4}(e_1 - e_2 - e_3 - e_4 - e_5 + e_6 + e_7 + 1) \\ \frac{1}{4}(-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{4}(-e_1 + e_2 - e_3 + e_4 - e_5 + e_6 - e_7 + 1) \\ \frac{1}{4}(-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{4}(-e_1 + e_2 - e_3 - e_4 + e_5 - e_6 + e_7 + 1) \\ \frac{1}{4}(-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{4}(-e_1 - e_2 + e_3 + e_4 - e_5 - e_6 + e_7 + 1) \\ \frac{1}{4}(-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{4}(-e_1 - e_2 + e_3 - e_4 + e_5 + e_6 - e_7 + 1) \\ \frac{1}{4}(-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{4}(e_1 + e_2 - e_3 + e_4 - e_5 - e_6 + e_7 - 1) \\ \frac{1}{4}(-e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 1) & \frac{1}{4}(e_1 + e_2 - e_3 - e_4 + e_5 + e_6 - e_7 - 1) \end{pmatrix}$$

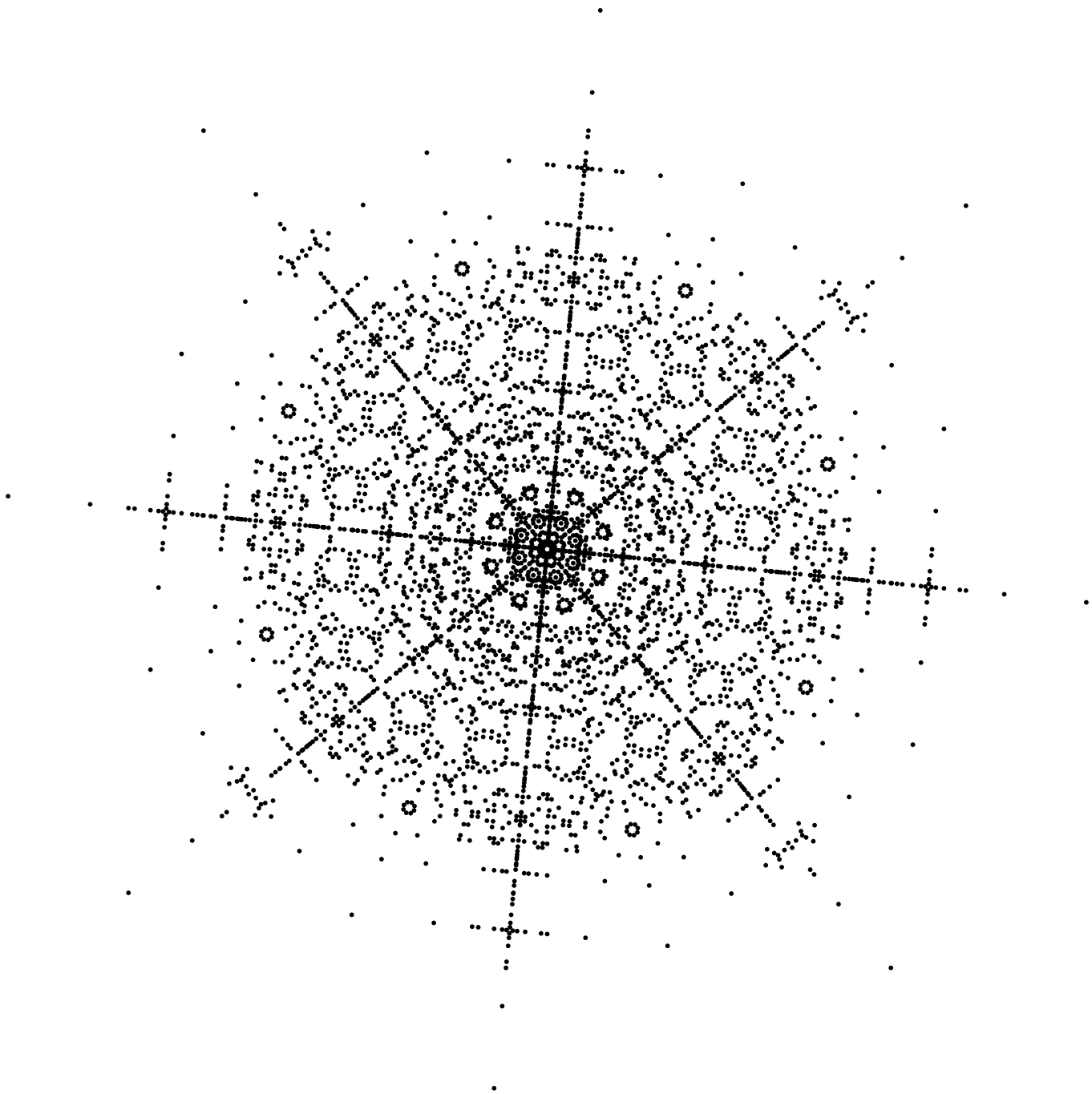

In[*]:= BW0a = Join[BW0480a, BW03840a];
Length@%

Out[*]=
480

(* Open this cell for the data *)
BW16Da = Flatten[oct2List@# & /@ #] & /@ BW0a;

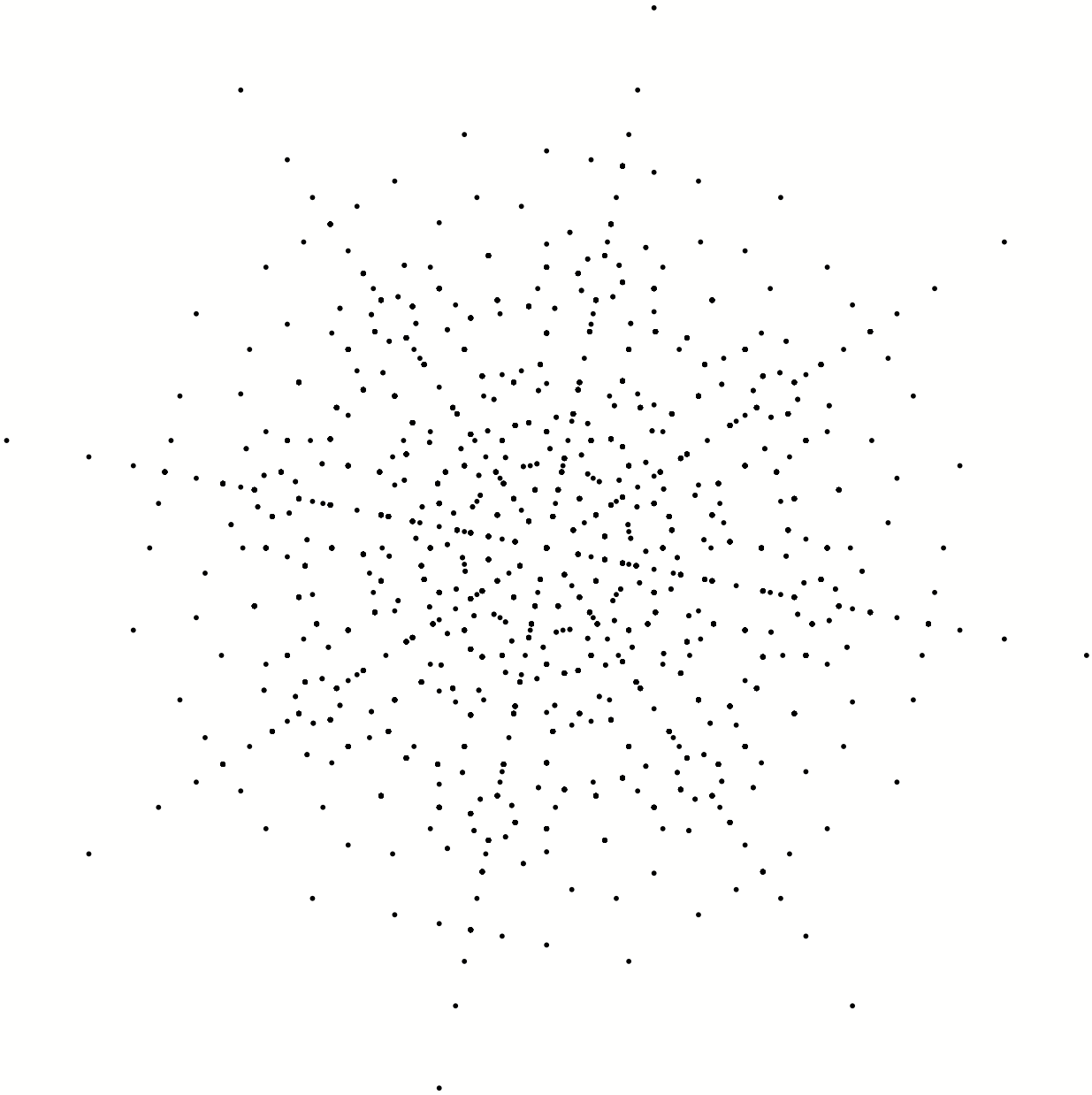
In[*]:= projBW16Da = Table[round[#, i], {i, (**) projBW16a(*) H430LR24**}] & /@ BW16Da;
Graphics@Point[projBW16Da[[All, ;; 2]]]
```

Out[*]=



```
In[*]:= projBW16D = Table[round[#,i], {i, (**)projBW16(*)H430LR24(**)}] & /@ BW16Da;  
Graphics@Point[projBW16D[[All, ;; 2]]]
```

Out[*]=



Set up the visualization of various Barnes-Wall & Leech Lattices

The output

Graphics@callImage

FactInt= $\begin{pmatrix} 3 & 1 \\ 7 & 1 \\ 29 & 1 \end{pmatrix}$, inc=203

Iteration=1 Start=0 Δ(s)=0

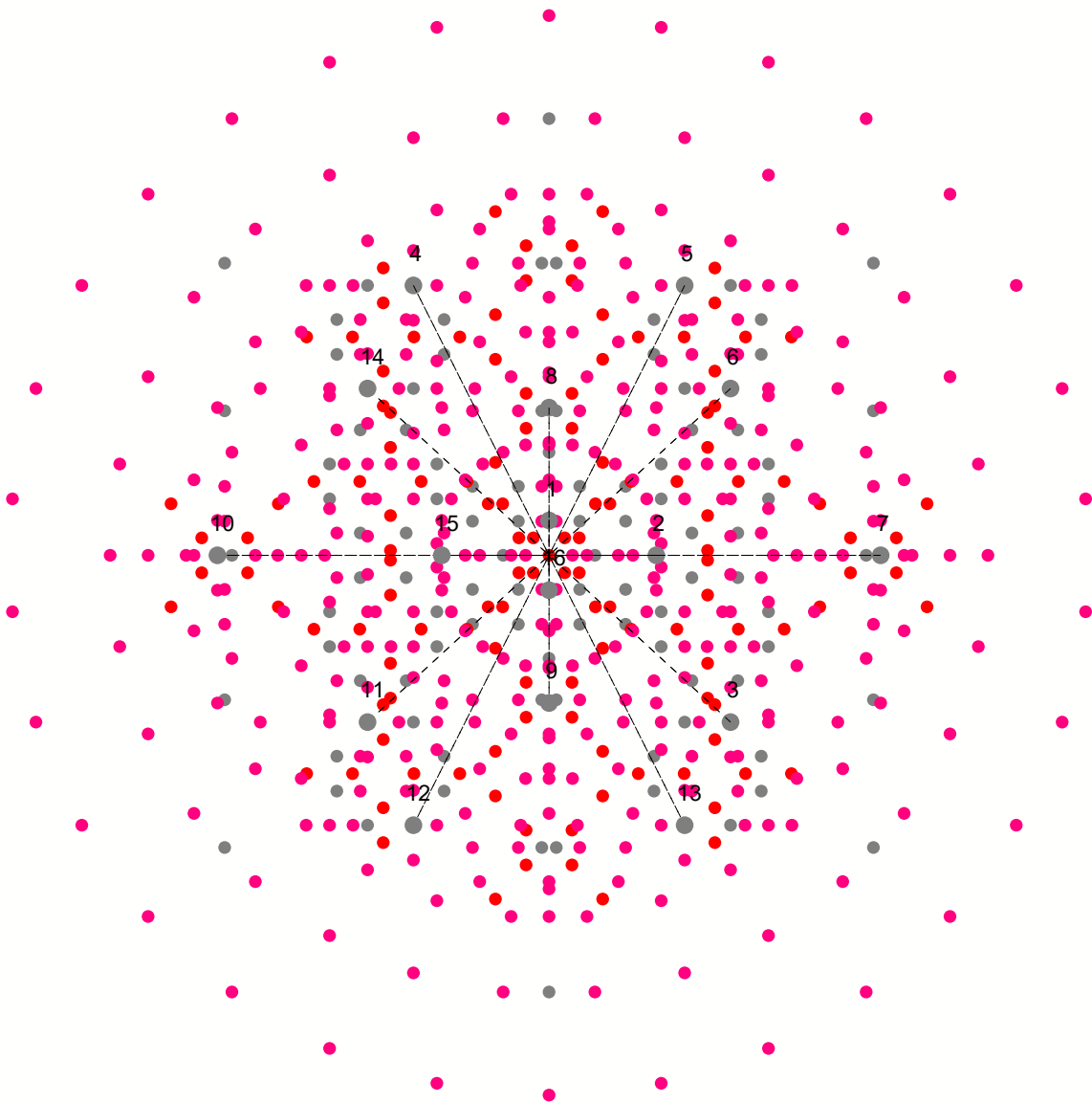
inc $\#$ =1 p $\#$ =1

Iteration=2 Start=29 Δ(s)=29

inc $\#$ =2 p $\#$ =204

Iteration=3 Start=53 Δ(s)=24

inc $\#$ =3 p $\#$ =407



vertices= 4320. edges= 0. triality= 0.
clicked= 0. step= 1. area= 0.00
faces= 0. cells= 0. volume= 0.00

In[*]:= vertexOverlaps

Out[*]//MatrixForm=

Edges=	0	Vertices=	4320
=====	=====	=====	=====
$\#$ of overlaps	$\#$ of vertices overlapping	assigned vertex color	$\#$ of visible vertices
-----	-----	-----	-----
1	240	■	240
2	256	■	128
14	1792	■	128
16	1792	■	112
240	240	■	1

$ln[*]:=$