

(in[1]:=) <-- Catalan Dual of A(111) Great Rhombicosidodecahedron (Disdyakis Triacanthedron)

Set up quaternion H3 Coxeter-Dynkin Weyl Orbit W(H3) (Kosz)

Kosz quaternion coordinate scales (See section 4.5 of <https://arxiv.org/abs/0908.3272>)

(out[1]:=) $\begin{aligned} i &= 3 \\ X1 &= \text{UnitVec}[1, \text{s1Rep}, 5] \\ 2 &= 3 \\ Y1 &= \text{UnitVec}[2, \text{s1Rep}, 3] \end{aligned}$

(in[1]:=) ABC triangle vertices as decagonal, hexagonal and the square great rhombicosidodecahedron face centers +

$A = X1 - \text{UnitVec}[e_1, \text{s1Rep}]$

$B = Y1 - \text{UnitVec}[e_1, \text{s1Rep}]$

$C = (e_1 - \text{UnitVec}[e_1, \text{s1Rep}]) / 2 - \text{UnitVec}[e_2, \text{s1Rep}]$

(in[1]:=) q is a point orthogonal to ABC +

$Q = (e_1 - 2 \cdot \text{UnitVec}[e_1, \text{s1Rep}]) / 2 / \sqrt{1 - \text{UnitVec}[e_1, \text{s1Rep}]^2}$

(in[1]:=) q' from q rotates by $\pi/5$ around A

q'' from q rotates by $\pi/5$ around A'

$Q'' = (2 \cdot e_1 - \sqrt{5} \cdot e_2 + (e_1 - e_2)) / 2 / \sqrt{1 - \text{UnitVec}[e_1, \text{s1Rep}]^2}$

(in[1]:=) r Circumradius of the Icosahedron +

$R = \text{UnitSimplify}@{\text{OctNorm}[A1]}$

$N = (\text{V}\{1\}, \text{muRep})$

$1/\epsilon$

(out[1]:=) $\frac{1}{\epsilon} \sqrt{\frac{(1-3 \sqrt{5})^2 (1-\varphi^2)}{\varphi^2}}$

(out[1]:=) 1.08876

(out[1]:=) 0.92101

(in[1]:=) \rightarrow Dodecahedron scaling +

$\text{Norm} = \text{FullSimplify}@\text{OctNorm}[0, 1, 1/\epsilon] / . \text{s1Rep}, \text{Assumptions} = \text{euAssumptions}$

$N/\epsilon, \text{muRep}$

$1/\epsilon^2$

(out[1]:=) $\frac{1}{\epsilon^2} \sqrt{\frac{(1-2 \sqrt{5})^2 (1-\varphi^2)}{\varphi^2}}$

(out[1]:=) 0.525731

(in[1]:=) \rightarrow Circumradius of the Dodecahedron +

$\text{Norm} = \text{FullSimplify}@\text{OctNorm}[0, 1]$

$N/\epsilon, \text{muRep}$

$1/\epsilon^2$

(out[1]:=) $\frac{1}{\epsilon^2} \sqrt{\frac{(1-2 \sqrt{5})^2 (1-\varphi^2)}{\varphi^2}}$

(out[1]:=) 0.101641

(out[1]:=) 0.981926

(in[1]:=) \rightarrow Dodecahedron scaling +

$\text{Norm} = \text{FullSimplify}@\text{Norm}[1, 1, 1]$

Norm/ϵ

$1/\epsilon$

(out[1]:=) $\frac{1}{\epsilon} \sqrt{\frac{1-2 \sqrt{5}}{\varphi}}$

(in[1]:=) \rightarrow Circumradius of the Icosidodecahedron +

$\text{Norm} = \text{FullSimplify}@\text{OctNorm}[0, 1]$

$N/\epsilon, \text{muRep}$

$1/\epsilon^2$

(out[1]:=) 1.

Original (incorrectly trying to correct Kosz & MTM coordinates using WP R) visualization of the Disdyakis Triacanthedron

with Icosahedron circumradius $\sqrt{3}$ 1Norm-9106 scaled Dodecahedron, S=Norm-92101 scaled (Dodecahedron)

3D Polygon angles for the ABC surface triangle for dVertxListKoca

(in[1]:=) FullSimplify[A1@P1]

$\text{Norm}/\epsilon, \text{muRep}$

$\text{ArcCos}[1]^*/$

$180 - \%$

DMSList@%

$(1-3 \varphi) \cdot (1-\varphi^2)$

(out[1]:=) $\frac{1}{\epsilon^2} \sqrt{\frac{(1-2 \sqrt{5})^2 (1-\varphi^2)}{\varphi^2}}$

(out[1]:=) 15.794654

(out[1]:=) 37.3774

(out[1]:=) 142.623

(out[1]:=) {142, 37, 21, 4747}

(in[1]:=) FullSimplify[A1@P2]

$\text{Norm}/\epsilon, \text{muRep}$

$\text{ArcCos}[1]^*/$

$180 - \%$

DMSList@%

$(1-3 \varphi) \cdot (1-\varphi^2)$

(out[1]:=) $\frac{1}{\epsilon^2} \sqrt{\frac{(1-2 \sqrt{5})^2 (1-\varphi^2)}{\varphi^2}}$

(out[1]:=) 0.859651

(out[1]:=) 31.7175

(out[1]:=) 148.283

(out[1]:=) {148, 16, 57, 0921}

(in[1]:=) FullSimplify[B1@C1]

$\text{Norm}/\epsilon, \text{muRep}$

$\text{ArcCos}[1]^*/$

$180 - \%$

DMSList@%

$(1-3 \varphi) \cdot (1-\varphi^2)$

(out[1]:=) $\frac{1}{\epsilon^2} \sqrt{\frac{(1-2 \sqrt{5})^2 (1-\varphi^2)}{\varphi^2}}$

(out[1]:=) 0.854172

(out[1]:=) 28.9052

(out[1]:=) 159.095

(out[1]:=) {159, 5, 41, 4332}

2D Polygon angles for the ABC surface triangle for dVertxListKoca

MTM Disdyakis Triacanthedron coordinates & visualization (uses scaled up and rotated original Kosz visualization coordinates)

MTM Norms are precisely the same as the Kosz Norms

Kosz Dihedral angle and AB, BC, AC angles from origin

Graphics3D[{{(*Dihedron0, 0, 0, 0), (*Dihedron1, 0, 0, 0)}, (*Dihedron2, 0, 0, 0), (*Dihedron3, 0, 0, 0), (*Dihedron4, 0, 0, 0), (*Dihedron5, 0, 0, 0), (*Dihedron6, 0, 0, 0), (*Dihedron7, 0, 0, 0), (*Dihedron8, 0, 0, 0), (*Dihedron9, 0, 0, 0), (*Dihedron10, 0, 0, 0), (*Dihedron11, 0, 0, 0), (*Dihedron12, 0, 0, 0), (*Dihedron13, 0, 0, 0), (*Dihedron14, 0, 0, 0), (*Dihedron15, 0, 0, 0), (*Dihedron16, 0, 0, 0), (*Dihedron17, 0, 0, 0), (*Dihedron18, 0, 0, 0), (*Dihedron19, 0, 0, 0), (*Dihedron20, 0, 0, 0), (*Dihedron21, 0, 0, 0), (*Dihedron22, 0, 0, 0), (*Dihedron23, 0, 0, 0), (*Dihedron24, 0, 0, 0), (*Dihedron25, 0, 0, 0), (*Dihedron26, 0, 0, 0), (*Dihedron27, 0, 0, 0), (*Dihedron28, 0, 0, 0), (*Dihedron29, 0, 0, 0), (*Dihedron30, 0, 0, 0), (*Dihedron31, 0, 0, 0), (*Dihedron32, 0, 0, 0), (*Dihedron33, 0, 0, 0), (*Dihedron34, 0, 0, 0), (*Dihedron35, 0, 0, 0), (*Dihedron36, 0, 0, 0), (*Dihedron37, 0, 0, 0), (*Dihedron38, 0, 0, 0), (*Dihedron39, 0, 0, 0), (*Dihedron40, 0, 0, 0), (*Dihedron41, 0, 0, 0), (*Dihedron42, 0, 0, 0), (*Dihedron43, 0, 0, 0), (*Dihedron44, 0, 0, 0), (*Dihedron45, 0, 0, 0), (*Dihedron46, 0, 0, 0), (*Dihedron47, 0, 0, 0), (*Dihedron48, 0, 0, 0), (*Dihedron49, 0, 0, 0), (*Dihedron50, 0, 0, 0), (*Dihedron51, 0, 0, 0), (*Dihedron52, 0, 0, 0), (*Dihedron53, 0, 0, 0), (*Dihedron54, 0, 0, 0), (*Dihedron55, 0, 0, 0), (*Dihedron56, 0, 0, 0), (*Dihedron57, 0, 0, 0), (*Dihedron58, 0, 0, 0), (*Dihedron59, 0, 0, 0), (*Dihedron60, 0, 0, 0), (*Dihedron61, 0, 0, 0), (*Dihedron62, 0, 0, 0), (*Dihedron63, 0, 0, 0), (*Dihedron64, 0, 0, 0), (*Dihedron65, 0, 0, 0), (*Dihedron66, 0, 0, 0), (*Dihedron67, 0, 0, 0), (*Dihedron68, 0, 0, 0), (*Dihedron69, 0, 0, 0), (*Dihedron70, 0, 0, 0), (*Dihedron71, 0, 0, 0), (*Dihedron72, 0, 0, 0), (*Dihedron73, 0, 0, 0), (*Dihedron74, 0, 0, 0), (*Dihedron75, 0, 0, 0), (*Dihedron76, 0, 0, 0), (*Dihedron77, 0, 0, 0), (*Dihedron78, 0, 0, 0), (*Dihedron79, 0, 0, 0), (*Dihedron80, 0, 0, 0), (*Dihedron81, 0, 0, 0), (*Dihedron82, 0, 0, 0), (*Dihedron83, 0, 0, 0), (*Dihedron84, 0, 0, 0), (*Dihedron85, 0, 0, 0), (*Dihedron86, 0, 0, 0), (*Dihedron87, 0, 0, 0), (*Dihedron88, 0, 0, 0), (*Dihedron89, 0, 0, 0), (*Dihedron90, 0, 0, 0), (*Dihedron91, 0, 0, 0), (*Dihedron92, 0, 0, 0), (*Dihedron93, 0, 0, 0), (*Dihedron94, 0, 0, 0), (*Dihedron95, 0, 0, 0), (*Dihedron96, 0, 0, 0), (*Dihedron97, 0, 0, 0), (*Dihedron98, 0, 0, 0), (*Dihedron99, 0, 0, 0), (*Dihedron100, 0, 0, 0), (*Dihedron101, 0, 0, 0), (*Dihedron102, 0, 0, 0), (*Dihedron103, 0, 0, 0), (*Dihedron104, 0, 0, 0), (*Dihedron105, 0, 0, 0), (*Dihedron106, 0, 0, 0), (*Dihedron107, 0, 0, 0), (*Dihedron108, 0, 0, 0), (*Dihedron109, 0, 0, 0), (*Dihedron110, 0, 0, 0), (*Dihedron111, 0, 0, 0), (*Dihedron112, 0, 0, 0), (*Dihedron113, 0, 0, 0), (*Dihedron114, 0, 0, 0), (*Dihedron115, 0, 0, 0), (*Dihedron116, 0, 0, 0), (*Dihedron117, 0, 0, 0), (*Dihedron118, 0, 0, 0), (*Dihedron119, 0, 0, 0), (*Dihedron120, 0, 0, 0), (*Dihedron121, 0, 0, 0), (*Dihedron122, 0, 0, 0), (*Dihedron123, 0, 0, 0), (*Dihedron124, 0, 0, 0), (*Dihedron125, 0, 0, 0), (*Dihedron126, 0, 0, 0), (*Dihedron127, 0, 0, 0), (*Dihedron128, 0, 0, 0), (*Dihedron129, 0, 0, 0), (*Dihedron130, 0, 0, 0), (*Dihedron131, 0, 0, 0), (*Dihedron132, 0, 0, 0), (*Dihedron133, 0, 0, 0), (*Dihedron134, 0, 0, 0), (*Dihedron135, 0, 0, 0), (*Dihedron136, 0, 0, 0), (*Dihedron137, 0, 0, 0), (*Dihedron138, 0, 0, 0), (*Dihedron139, 0, 0, 0), (*Dihedron140, 0, 0, 0), (*Dihedron141, 0, 0, 0), (*Dihedron142, 0, 0, 0), (*Dihedron143, 0, 0, 0), (*Dihedron144, 0, 0, 0), (*Dihedron145, 0, 0, 0), (*Dihedron146, 0, 0, 0), (*Dihedron147, 0, 0, 0), (*Dihedron148, 0, 0, 0), (*Dihedron149, 0, 0, 0), (*Dihedron150, 0, 0, 0), (*Dihedron151, 0, 0, 0), (*Dihedron152, 0, 0, 0), (*Dihedron153, 0, 0, 0), (*Dihedron154, 0, 0, 0), (*Dihedron155, 0, 0, 0), (*Dihedron156, 0, 0, 0), (*Dihedron157, 0, 0, 0), (*Dihedron158, 0, 0, 0), (*Dihedron159, 0, 0, 0), (*Dihedron160, 0, 0, 0), (*Dihedron161, 0, 0, 0), (*Dihedron162, 0, 0, 0), (*Dihedron163, 0, 0, 0), (*Dihedron164, 0, 0, 0), (*Dihedron165, 0, 0, 0), (*Dihedron166, 0, 0, 0), (*Dihedron167, 0, 0, 0), (*Dihedron168, 0, 0, 0), (*Dihedron169, 0, 0, 0), (*Dihedron170, 0, 0, 0), (*Dihedron171, 0, 0, 0), (*Dihedron172, 0, 0, 0), (*Dihedron173, 0, 0, 0), (*Dihedron174, 0, 0, 0), (*Dihedron175, 0, 0, 0), (*Dihedron176, 0, 0, 0), (*Dihedron177, 0, 0, 0), (*Dihedron178, 0, 0, 0), (*Dihedron179, 0, 0, 0), (*Dihedron180, 0, 0, 0), (*Dihedron181, 0, 0, 0), (*Dihedron182, 0, 0, 0), (*Dihedron1