Cohl Furey’s ℝ⊗ℂ⊗ℍ⊗𝕆 papers:

A Computational Analysis by J. Gregory Moxness

Motivation

This is an updated analysis of several papers from Cohl Furey related to using complexified real octonions (aka. bi-octonions) in the assignment of Standard Model (SM) Boson and Fermion particles to Clifford algebras and group symmetries. The previous analysis was based on [1-4]:

[1] https://arxiv.org/abs/1002.1497v5

[2] https://arxiv.org/abs/1405.4601v1

[3] https://arxiv.org/abs/1603.04078

[4] https://arxiv.org/abs/1806.00612

[5] https://arxiv.org/abs/1910.08395v1

This .pdf is a direct output from my Mathematica (MTM) Notebook. I will follow up with a LaTex paper on the topic soon.

This notebook has code built in to operate symbolically on native MTM reals (ℝ), complexes (ℂ=a+ⅈb), and quaternionic (ℍ=a+ⅈb+cⅉ+dk) forms, as well as my custom code to handle the octonions (𝕆 ={e_0 = 1, e_1 = ⅈ, e_2 = ⅉ, e_3 = k, e_4, e_5, e_6, e_7}), and now the bi-octonions (which doesn't assign the octonion e_1 to be equivalent to the complex imaginary (ⅈ)). That change also applies to the native quaternion assignments where of e_1 = i, e_2 = j, e_3 = k) in order to work with quater-octonions. This was a fairly trivial change to make since it simply involves removing the conversion of complex (and quaternion) operators from being involved in the octonionic multiplication (i.e. SmallCircle[] or symbolically ∘).

Commenting out that code is shown here:

Please note that my previous analysis here made the mistake of not commenting out these operations. As such, it was operating on octonions (not complexified bi-octonions), so some of my concerns were resolved based on correcting that error.

This also creates a split in the way conjugation is handled. The MTM native conjugation (\[Conjugate] or *) handles the imaginary components of ℝ, ℂ, and ℍ (after loading <<Quaternions`). With the change above, my code operates on octonion symbols (i.e. oct = {e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7}) as real as required for bi-octonions, but my octonion conjugation (octCongugate[] or $x^{*(\text{asterisk})}$) computation operates on octonion symbols as imaginary in addition to the imaginary ℂ∊ℍ elements if present (i.e. $ⅈ\, e_n \to (-ⅈ)\,(-e_n) = ⅈ\, e_n$).

The following code implements the two forms of conjugation introduced in the papers (* and †):

I’ve set up this “bi-octonion ℝ⊗ℂ⊗ℍ⊗𝕆 machine” to easily switch from native MTM conjugation and my octonion conjugation.

The exceptions for s & S in this code accommodates their use in these papers as complex elements that only get MTM native treatment, thus avoiding recursive loops when not being reduced with used with the real octonion elements).

The following code implements the s and S in [5]:

In[2197]:=
```
doConjugate = False;
```

In[2198]:=
```
(* Conjugate *)
sScon = {
   {s*, S},
   {s, S*},
   {s*, S*},
   {s, S}};
```

In[2199]:=

It is also interesting to note that unlike complexes (ℂ), there are actually two forms of quaternion (ℍ) assignments where 𝕜=(𝕚𝕛 or 𝕛𝕚). For octonions, there are 480 possible multiplication tables that are non-trivial (i.e. not simple permutations of the oct symbols). There are 3840 split octonion multiplication tables. These multiplication tables are generated in my code using any of the algorithmically generated 480 possible octonion multiplication tables, as well as visualizing their Fano plane (or Fano cube) mnemonic based on which of the 7 triads selected. This code is also built to accommodate sedenions by Cayley-Dickson doubling, but I digress.

I am very interested here in the suggestion at the very end of Cohl's last paper [5] in the Addendum Section IX(B/C) on Multi-actions splitting spinor spaces, Lie algebras/groups, and Jordan algebras. I suspect having the ability to create a machine (i.e. a symbolic engine such as MTM) to operate on and visualize these structures as hyper-dimensional physical elements is critical to making progress in understanding our Universe more thoroughly.

From [1] Towards a unified theory of ideals (2010)

Select and implement the specific octonion:

So to set up the analysis, we select the Fano plane used in these papers, which happens to be the same on used by Baez in his work, which is in position 224 "flip→True" in my assignment of the 480 combinations. The algorithm to visualize the Fano plane constructs it differently than the paper, but the triads are correct.

In[2204]:=
```
(* Find Cohl Furey's Octonion = Baez's octonion based on triads *)
Position[(setFM[#, True];
    StandardForm[triads]) & /@ Range@256,
 {{1, 2, 4}, {1, 3, 7}, {1, 5, 6}, {2, 3, 5}, {2, 6, 7}, {3, 4, 6}, {4, 5, 7}}]
```

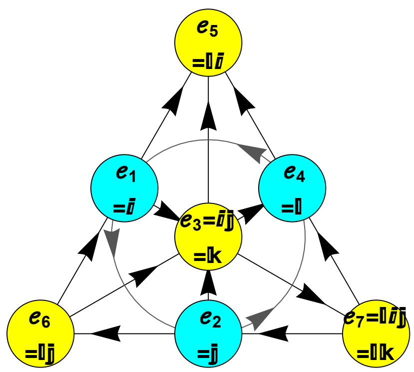
Out[2204]= (224 1)

In[2205]:=
```
(* Set the Fano Multiplication matrix to that triad set (224, flip=True)
 and display the triads, Fano plane and multiplication table *)
setFM[224, flip = True];
Row@{triads, fanoPlane,
  IJKLstyle = False; fmDispe,
  IJKLstyle = True;
  fmDispe}
```

Out[2206]=

1	2	4
1	3	7
1	5	6
2	3	5
2	6	7
3	4	6
4	5	7

Octonion Fano Plane



$e_0\mapsto 1$	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	-1	e_4	e_7	$-e_2$	e_6	$-e_5$	$-e_3$
e_2	$-e_4$	-1	e_5	e_1	$-e_3$	e_7	$-e_6$
e_3	$-e_7$	$-e_5$	-1	e_6	e_2	$-e_4$	e_1
e_4	e_2	$-e_1$	$-e_6$	-1	e_7	e_3	$-e_5$
e_5	$-e_6$	e_3	$-e_2$	$-e_7$	-1	e_1	e_4
e_6	e_5	$-e_7$	e_4	$-e_3$	$-e_1$	-1	e_2
e_7	e_3	e_6	$-e_1$	e_5	$-e_4$	$-e_2$	-1

$e_0\mapsto 1$	𝕚	𝕛	𝕜	𝕝	𝕝𝕚	𝕝𝕛	𝕝𝕜
𝕚	-1	𝕝	𝕝𝕜	−𝕛	𝕝𝕛	−𝕝𝕚	−𝕜
𝕛	−𝕝	-1	𝕝𝕚	𝕚	−𝕜	𝕝𝕜	−𝕝𝕛
𝕜	−𝕝𝕜	−𝕝𝕚	-1	𝕝𝕛	𝕛	−𝕝	𝕚
𝕝	𝕛	−𝕚	−𝕝𝕛	-1	𝕝𝕜	𝕜	−𝕝𝕚
𝕝𝕚	−𝕝𝕛	𝕜	−𝕛	−𝕝𝕜	-1	𝕚	𝕝
𝕝𝕛	𝕝𝕚	−𝕝𝕜	𝕝	−𝕜	−𝕚	-1	𝕛
𝕝𝕜	𝕜	𝕝𝕛	−𝕚	𝕝𝕚	−𝕝	−𝕛	-1

It should be noted that there can be risk in making assumptions based on operating on a given set of triads→ Fano planes→ multiplication tables. Some of the patterns are specific only to that selected octonion out of the 480 (e.g. A reference in [2] from a reference to the Baez's paper (https://arxiv.org/abs/math/0105155) suggesting that index cycling and doubling patterns hold. While interesting, this is only true for that specific octonion multiplication table used in these papers.

Cohl's papers do not reduce the octonionic products (i.e. $e_1 \circ e_2 = e_{12} = e_4$) based on the selected multiplication table outside of using identities common to all imaginary numbers (e.g. $e_n \circ e_m = e_{nm}$, $e_{nm} = -e_{mn}$, and $e_n \circ e_n = -1$).

The following code implements functions needed for basic octonion processing (with some new code for bi-octonions):

Beginning the introduction of octonion chains including octonion reduction using the selected triad multiplication table.

In[2214]:=
```
(* Some simple octProduct math *)
```

In[2215]:= $e_1 \circ e_2$

Out[2215]= e_4

In[2216]:= $e_2 \circ e_3$

Out[2216]= e_5

In[2217]:= $e_1 \circ (e_2 \circ e_3)$

Out[2217]= e_6

In[2218]:=
```
e3 ∘ (e2 ∘ e1)
```
Out[2218]= $-e_6$

In[2219]:=
```
e6 + ⅈ ∘ e2
```
Out[2219]= $e_4 + e_6$

Implementing Left vs. Right chains and precedence mapping function (f):

I constructed a function f to build the left (L) parenthetical multiplication precedence, which can be reversed by simply providing a “right→True” parameter to perform the right (R) operations. In addition, it takes a “right→both” parameter to both reverse the order and the parenthetical precedence. Note the use of the SmallCircle "∘" for doing proper octonion multiplication. While the paper does not show the evaluation of the octonionic multiplication, I do that in order to verify the logic.

In[2220]:=
```
(* Define the parenthetical reverse nesting with Octonion multiplication table operation ∘ *)
Clear@f;
f[in_List, right_: False] := Module[{inFlat = Flatten@in, out = 1},
   Do[out = If[right === both, inFlat⟦l⟧ ∘ out,
      If[right, out ∘ inFlat⟦l⟧,
       Reverse[inFlat]⟦l⟧ ∘ out]], {l, Length@inFlat}];
   Simplify@out];
```

Using an example from [2] Generations: Three Prints, in Colour (2014) paragraph “Octonionic Chains”, we evaluate this function first w/o a defined octonion product table to see the proper parenthetical precedence is obtained for left and right operations:

In[2222]:=
```
Clear[SmallCircle]
```

In[2223]:=
```
f@{e3, e4, e6 + ⅈ ∘ e2}
```
Out[2223]= $e_3 \circ (e_4 \circ ((e_6 + i \circ e_2) \circ 1))$

Redefine SmallCircle and re-evaluate:

In[2224]:=
```
SmallCircle[a_, 1] := a;
SmallCircle[1, b_] := b;
SmallCircle[a_List, b_List] := octonion@octProduct[a, b];
SmallCircle[a_, b_] := If[chksS@{a, b}, a b, oct2List@a ∘ oct2List@b];
```

In[2228]:=
```
f@{e3, e4, e6 + ⅈ ∘ e2}
(* or in Mathematica postfix notation as a right to left functor as used in the paper *)
{e3, e4, e6 + ⅈ ∘ e2} // f
```
Out[2228]= $-1 + i\, e_7$

Out[2229]= $-1 + i\, e_7$

The associator function:

In[2230]:=
```
associator[a_List, b_List, c_List] := (a ∘ b) ∘ c - a ∘ (b ∘ c);
associator[a_, b_, c_] := associator[oct2List@a, oct2List@b, oct2List@c];
```

If the associator is zero, it doesn’t show the non-associative property.

In[2232]:=
```
associator[e3, e4, e5]
```
Out[2232]= $-2\, e_1$

In[2233]:=
```
associator[e3, e4, e6]
```
Out[2233]= 0

From [2]: Generations: Three Prints, in Colour (2014)

The following implements / checks the ν identities:

In[2234]:=
```
ν = -1/2 f[{e3, e4, e6 + ⅈ∘e2}, True]
```
Out[2234]= $\frac{1}{2}(1 + i\, e_7)$

In[2235]:=
```
(* Normal Conjugation - only negates ⅈ without changing en *)
ν*
```
Out[2235]= $\frac{1}{2}(1 - i\, e_7)$

In[2236]:=
```
(* Cross Conjugation -
 negates the real compared to normal (or both compared to octConjugation *)
ν†
```
Out[2236]= $-\frac{1}{2} - \frac{i\, e_7}{2}$

In[2237]:=
```
(* octConjugation - negates both ⅈ and en resulting in no change on ⅈ en *)
ν*
```
Out[2237]= $\frac{1}{2} + \frac{i\, e_7}{2}$

In[2238]:=
```
(* No Conjugation Identities *)
Simplify[1/2 (1 + ⅈ e7) == ν == ν∘ν]
```
Out[2238]= True

In[2239]:=
```
(* octConjugation Identities *)
Simplify[1/2 (1 + ⅈ e7) == ν* == ν*∘ν*]
```
Out[2239]= True

In[2240]:=
```
(* Normal Conjugation Identities *)
Simplify[1/2 (1 - ⅈ e7) == ν* == ν*∘ν*]
0 == ν∘ν* == ν*∘ν
```
Out[2240]= True

Out[2241]= True

In[2242]:=
```
(* Cross Conjugation Identities *)
```

In[2243]:=
```
ν†
```
Out[2243]= $-\frac{1}{2} - \frac{i\, e_7}{2}$

In[2244]:=
```
ν†∘ν†
```
Out[2244]= $\frac{1}{2} + \frac{i\, e_7}{2}$

In[2245]:=
```
(* Cross Conjugation Identities *)
```
$$\frac{1}{2} + \frac{i\, e_7}{2} == -\nu^\dagger == \nu^\dagger \circ \nu^\dagger$$
$$\nu^\dagger == \nu \circ \nu^\dagger == \nu^\dagger \circ \nu$$

Out[2245]= True

Out[2246]= True

In[2247]:= $\nu \odot \nu^\dagger$

Out[2247]= 0

From [3]: Charge quantization from a number operator (2016)

The following implements / checks the new basis {$\alpha1$, $\alpha2$, $\alpha3$}:

In[2248]:=
```
Clear[α, α1, α2, α3]
```

In[2249]:= $\alpha1 = \frac{1}{2}\left(-e_5 + i\, e_4\right)$

Out[2249]= $\frac{1}{2}(-e_5 + i\, e_4)$

In[2250]:=
```
(* Normal Conjugation *)
```
$\alpha1^*$

Out[2250]= $\frac{1}{2}(-e_5 - i\, e_4)$

In[2251]:=
```
(* octConjugation *)
```
$\alpha1^*$

Out[2251]= $\frac{e_5}{2} + \frac{i\, e_4}{2}$

In[2252]:=
```
(* Cross Conjugation *)
```
$\alpha1^\dagger$

Out[2252]= $-\frac{e_5}{2} - \frac{i\, e_4}{2}$

In[2253]:= $\alpha2 = \frac{1}{2}\left(-e_3 + i\, e_1\right)$

Out[2253]= $\frac{1}{2}(-e_3 + i\, e_1)$

In[2254]:=
```
(* Normal Conjugation *)
```
$\alpha2^*$

Out[2254]= $\frac{1}{2}(-e_3 - i\, e_1)$

In[2255]:= $\alpha2^*$

Out[2255]= $\frac{e_3}{2} + \frac{i\, e_1}{2}$

In[2256]:= $\alpha2^\dagger$

Out[2256]= $-\frac{e_3}{2} - \frac{i\, e_1}{2}$

In[2257]:= $\alpha3 = \frac{1}{2}\left(-e_6 + i\, e_2\right)$

Out[2257]= $\frac{1}{2}(-e_6 + i\, e_2)$

In[2258]:=
```
(* Normal Conjugation *)
α3*
```
Out[2258]= $\frac{1}{2}(-e_6 - i\, e_2)$

In[2259]:=
```
α3*
```
Out[2259]= $\frac{e_6}{2} + \frac{i\, e_2}{2}$

In[2260]:=
```
α3†
```
Out[2260]= $-\frac{e_6}{2} - \frac{i\, e_2}{2}$

In[2261]:=
```
Simplify[f@{α1, α2} == -i α3†]
Simplify[f@{α3, α1} == -i α2†]
Simplify[f@{α2, α3} == -i α1†]
```
Out[2261]= True

Out[2262]= True

Out[2263]= True

Applying f and octonion multiplication reduces the following to zero:

In[2264]:=
```
Simplify@{-{α2†, α1}, -{α1†, α2}}
{-f@{α2†, α1}, -f@{α1†, α2}}
```
Out[2264]= $\begin{pmatrix} \frac{1}{2}(i\, e_1 + e_3) & \frac{1}{2}(e_5 - i\, e_4) \\ \frac{1}{2}(i\, e_4 + e_5) & \frac{1}{2}(e_3 - i\, e_1) \end{pmatrix}$

Out[2265]= {0, 0}

In[2266]:=
```
{#[[1]]∘#[[2]]} & /@ {{α1, α2}, {α2, α3}, {α1, α3}}
{#[[1]]†∘#[[2]]†} & /@ {{α1, α2}, {α2, α3}, {α1, α3}}
{#[[1]], #[[2]]†, #[[1]]∘#[[2]]†, #[[2]]∘#[[1]]†} & /@ {{α1, α2}, {α2, α3}, {α1, α3}}
```
Out[2266]= $\begin{pmatrix} \frac{i\, e_6}{2} - \frac{e_2}{2} \\ \frac{i\, e_5}{2} - \frac{e_4}{2} \\ \frac{e_1}{2} - \frac{i\, e_3}{2} \end{pmatrix}$

Out[2267]= $\begin{pmatrix} -\frac{e_2}{2} - \frac{i\, e_6}{2} \\ -\frac{e_4}{2} - \frac{i\, e_5}{2} \\ \frac{e_1}{2} + \frac{i\, e_3}{2} \end{pmatrix}$

Out[2268]= $\begin{pmatrix} \frac{1}{2}(i\, e_4 - e_5) & -\frac{i\, e_1}{2} - \frac{e_3}{2} & 0 & 0 \\ \frac{1}{2}(i\, e_1 - e_3) & -\frac{i\, e_2}{2} - \frac{e_6}{2} & 0 & 0 \\ \frac{1}{2}(i\, e_4 - e_5) & -\frac{i\, e_2}{2} - \frac{e_6}{2} & 0 & 0 \end{pmatrix}$

The following implements / checks ω identities:

In[2269]:=
```
(* ω (or is it ν) is a negative charge particle that converts up →
 down via right multiplication (i.e. W-) *)
ω = f@{α1, α2, α3}
```
Out[2269]= $\frac{1}{2}(-e_7 + i)$

In[2270]:=
```
(* octConjugation *)
ω*
```
Out[2270]= $\frac{e_7}{2} - \frac{i}{2}$

In[2271]:=
```
(* Normal Conjugation *)
ω*
```
Out[2271]= $\frac{1}{2}(-e_7 - i)$

In[2272]:=
```
ω†
```
Out[2272]= $-\frac{e_7}{2} + \frac{i}{2}$

In[2273]:=
```
(* ω† (or is it ν*) is a positive charge particle that converts down →
 up via right multiplication (i.e. W+) *)
Expand[ω == ω†]
```
Out[2273]= True

In[2274]:=
```
f@{α1†, α2†, α3†}
```
Out[2274]= $\frac{1}{2}(-e_7 - i)$

In[2275]:=
```
ν = Simplify[-f@{ω, ω†}]
```
Out[2275]= $\frac{1}{2}(1 + i\, e_7)$

In[2276]:=
```
Expand[-ν == f@{ω, ω†} == f@{ω†, ω} == ν†]
```
Out[2276]= True

In[2277]:=
```
{ToString[Subscript["2α", #], TraditionalForm] <>
    "=" <> ToString[Expand[2 α#], TraditionalForm], Spacer@10,
   ToString[Subsuperscript["2α", #, "†"], TraditionalForm] "=" <>
     ToString[Subsuperscript["2α", #, "*"], TraditionalForm] <> "=", Expand[2 α#†], Spacer@10,
   ToString[Subsuperscript["2α", #, "oct*"], TraditionalForm] <>
    "=" <> ToString[Expand[2 α#*], TraditionalForm]} & /@ Range@3
```
Out[2277]=

$2\alpha_1{=}2\,\alpha_1$	$2\alpha_1^{\dagger}{=}2\alpha_1^{*}{=}$	$2\,e_1$	$2\alpha_1^{oct*}{=}{-}2\,e_1$
$2\alpha_2{=}2\,\alpha_2$	$2\alpha_2^{\dagger}{=}2\alpha_2^{*}{=}$	$2\,e_2$	$2\alpha_2^{oct*}{=}{-}2\,e_2$
$2\alpha_3{=}2\,\alpha_3$	$2\alpha_3^{\dagger}{=}2\alpha_3^{*}{=}$	$2\,e_3$	$2\alpha_3^{oct*}{=}{-}2\,e_3$

In[2278]:=
```
ω† ∘ α#† & /@ Range@3
```
Out[2278]= $\left\{-\frac{e_3}{2} + \frac{i\, e_1}{2}, -\frac{e_6}{2} + \frac{i\, e_2}{2}, \frac{e_1}{2} + \frac{i\, e_3}{2}\right\}$

In[2279]:=
```
α# ∘ ν & /@ Range@3
```
Out[2279]= $\left\{\frac{e_1}{2} - \frac{i\, e_3}{2}, \frac{e_2}{2} - \frac{i\, e_6}{2}, \frac{e_3}{2} + \frac{i\, e_1}{2}\right\}$

The following implements / checks the new basis $\Lambda_1 - \Lambda_8$:

Commutation (x⊙y) and AntiCommutation (x⊖y)

In[2302]:=
```
commutator[x_, y_] := Simplify@Chop[x∘y - y∘x] /. slRep
If[! doClifford,
   CircleDot[x_, y_] := commutator[x, y];
   derivation[x_, y_][a_] := (x⊙y) ⊙a - 3 associator[x, y, a],
   derivation[x_, y_][a_] := commutator[commutator[x, y], a] - 3 associator[x, y, a]] /. slRep;
```

In[2304]:=
```
antiCommutator[x_, y_] := Simplify@Chop[x∘y + y∘x] /. slRep;
CircleMinus[x_, y_] := antiCommutator[x, y];
```

Commutation checks

In[2306]:= $\sum_{j=2}^{6} \sum_{i=1}^{j-1}$ f@oct[[{i, j}]] ⊙ f@oct[[{j, i}]]

Out[2306]= 0

In[2307]:= $\sum_{j=2}^{6} \sum_{i=1}^{j-1}$ (f@oct[[{i, j}]] + f@oct[[{j, i}]])

Out[2307]= 0

AntiCommutation checks

In[2308]:= α1 ⊖ α1

Out[2308]= 0

In[2309]:= α1 ⊖ $\alpha 2^{\dagger}$

Out[2309]= 0

In[2310]:= α1 ⊖ $\alpha 1^{\dagger}$

Out[2310]= −1

From [4]: $(SU(3))_C \times (SU(2))_L \times (U(1))_Y \times (U(1))_x$ as a symmetry of division algebraic ladder operators (2018)

4.3 Quarks and leptons as minimal left ideals (A one generation SM)

From [5] : Three generations, two unbroken gauge symmetries, and one eight-dimensional algebra (2019)

4.2 Clifford algebraic structure (Mf)

The following visualizes the 0-V to 6-V derived from MTM subsets:

In[2404]:=
```
gather = GatherBy[Subsets@Range@6, Length@# &]
Length /@ %
```

Out[2404]=

$$\left\{\{\{\}\}, \begin{pmatrix}1\\2\\3\\4\\5\\6\end{pmatrix}, \begin{pmatrix}1&2\\1&3\\1&4\\1&5\\1&6\\2&3\\2&4\\2&5\\2&6\\3&4\\3&5\\3&6\\4&5\\4&6\\5&6\end{pmatrix}, \begin{pmatrix}1&2&3\\1&2&4\\1&2&5\\1&2&6\\1&3&4\\1&3&5\\1&3&6\\1&4&5\\1&4&6\\1&5&6\\2&3&4\\2&3&5\\2&3&6\\2&4&5\\2&4&6\\2&5&6\\3&4&5\\3&4&6\\3&5&6\\4&5&6\end{pmatrix}, \begin{pmatrix}1&2&3&4\\1&2&3&5\\1&2&3&6\\1&2&4&5\\1&2&4&6\\1&2&5&6\\1&3&4&5\\1&3&4&6\\1&3&5&6\\1&4&5&6\\2&3&4&5\\2&3&4&6\\2&3&5&6\\2&4&5&6\\3&4&5&6\end{pmatrix}, \begin{pmatrix}1&2&3&4&5\\1&2&3&4&6\\1&2&3&5&6\\1&2&4&5&6\\1&3&4&5&6\\2&3&4&5&6\end{pmatrix}, \begin{pmatrix}1&2&3&4&5&6\end{pmatrix}\right\}$$

Out[2405]= {1, 6, 15, 20, 15, 6, 1}

In[2406]:=
```
invGather = (Complement[Range@7, #] & /@ #) & /@ gather
```

Out[2406]=

$$\left\{\begin{pmatrix}1&2&3&4&5&6&7\end{pmatrix}, \begin{pmatrix}2&3&4&5&6&7\\1&3&4&5&6&7\\1&2&4&5&6&7\\1&2&3&5&6&7\\1&2&3&4&6&7\\1&2&3&4&5&7\end{pmatrix}, \begin{pmatrix}3&4&5&6&7\\2&4&5&6&7\\2&3&5&6&7\\2&3&4&6&7\\2&3&4&5&7\\1&4&5&6&7\\1&3&5&6&7\\1&3&4&6&7\\1&3&4&5&7\\1&2&5&6&7\\1&2&4&6&7\\1&2&4&5&7\\1&2&3&6&7\\1&2&3&5&7\\1&2&3&4&7\end{pmatrix}, \begin{pmatrix}4&5&6&7\\3&5&6&7\\3&4&6&7\\3&4&5&7\\2&5&6&7\\2&4&6&7\\2&4&5&7\\2&3&6&7\\2&3&5&7\\2&3&4&7\\1&5&6&7\\1&4&6&7\\1&4&5&7\\1&3&6&7\\1&3&5&7\\1&3&4&7\\1&2&6&7\\1&2&5&7\\1&2&4&7\\1&2&3&7\end{pmatrix}, \begin{pmatrix}5&6&7\\4&6&7\\4&5&7\\3&6&7\\3&5&7\\3&4&7\\2&6&7\\2&5&7\\2&4&7\\2&3&7\\1&6&7\\1&5&7\\1&4&7\\1&3&7\\1&2&7\end{pmatrix}, \begin{pmatrix}6&7\\5&7\\4&7\\3&7\\2&7\\1&7\end{pmatrix}, \begin{pmatrix}7\end{pmatrix}\right\}$$

In[2407]:=
```
(* Now show these as reduced bi-octonion elements L/R→Bottom/Top *)
addI = (*)False**)True(**);
Table[{
   If[addI && EvenQ@i, ⅈ, e0] ∘ f@oct⟦gather⟦i, j⟧⟧,
   If[addI && EvenQ@i, ⅈ, e0] ∘ f@oct⟦invGather⟦i, j⟧⟧}, {i, 7}, {j, Length@gather⟦i⟧}] /. e0 → 1
Reverse[Total@# & /@ %]
(*)
     Total@#&/@%//ColumnForm
   Total@%//ColumnForm**)
```

Out[2408]=

$$\left\{\begin{pmatrix}1 & -1\end{pmatrix}, \begin{pmatrix} i e_1 & i e_1 \\ i e_2 & -i e_2 \\ i e_3 & i e_3 \\ i e_4 & -i e_4 \\ i e_5 & i e_5 \\ i e_6 & -i e_6 \end{pmatrix}, \begin{pmatrix} e_4 & e_4 \\ e_7 & -e_7 \\ -e_2 & -e_2 \\ e_6 & -e_6 \\ -e_5 & -e_5 \\ e_5 & e_5 \\ e_1 & -e_1 \\ -e_3 & -e_3 \\ e_7 & -e_7 \\ e_6 & e_6 \\ e_2 & -e_2 \\ -e_4 & -e_4 \\ e_7 & e_7 \\ e_3 & -e_3 \\ e_1 & e_1 \end{pmatrix}, \begin{pmatrix} i e_6 & -i e_6 \\ -i & -i \\ -i e_7 & i e_7 \\ -i e_3 & -i e_3 \\ -i e_5 & i e_5 \\ i e_4 & i e_4 \\ i e_2 & -i e_2 \\ -i e_3 & i e_3 \\ i e_7 & i e_7 \\ -i & i \\ i e_7 & i e_7 \\ -i & i \\ -i e_1 & -i e_1 \\ -i e_6 & -i e_6 \\ i e_5 & -i e_5 \\ -i e_4 & -i e_4 \\ i e_1 & -i e_1 \\ -i & -i \\ -i e_7 & i e_7 \\ i e_2 & i e_2 \end{pmatrix}, \begin{pmatrix} -e_3 & e_3 \\ -e_1 & -e_1 \\ 1 & -1 \\ e_5 & -e_5 \\ e_6 & e_6 \\ e_2 & -e_2 \\ -1 & -1 \\ -e_1 & e_1 \\ e_3 & e_3 \\ e_4 & -e_4 \\ -e_4 & e_4 \\ -e_2 & -e_2 \\ e_6 & -e_6 \\ -1 & -1 \\ -e_5 & e_5 \end{pmatrix}, \begin{pmatrix} i e_2 & i e_2 \\ -i e_4 & i e_4 \\ -i e_5 & -i e_5 \\ -i e_1 & i e_1 \\ -i e_6 & -i e_6 \\ i e_3 & -i e_3 \end{pmatrix}, \begin{pmatrix} e_7 & e_7 \end{pmatrix}\right\}$$

Out[2409]=

$$\begin{pmatrix} e_7 & e_7 \\ -i e_1 + i e_2 + i e_3 - i e_4 - i e_5 - i e_6 & i e_1 + i e_2 - i e_3 + i e_4 - i e_5 - i e_6 \\ -2 e_1 + 2 e_6 - 1 & -2 e_2 + 2 e_3 - 3 \\ 2 i e_2 - 2 i e_3 - 4 i & -2 i e_1 - 2 i e_6 + 4 i e_7 \\ 2 e_1 + 2 e_6 + 3 e_7 & -2 e_2 - 2 e_3 - e_7 \\ i e_1 + i e_2 + i e_3 + i e_4 + i e_5 + i e_6 & i e_1 - i e_2 + i e_3 - i e_4 + i e_5 - i e_6 \\ 1 & -1 \end{pmatrix}$$

Notice that the symmetry of the structure is more complicated than before applying the octonion Product.

Associativity of Composition of Maps

In[2410]:=
```
Clear[F, G, H]
```

In[2413]:=
```
F := f@{e2, e3, #} /. f → fff &;
G := f@{e3, #} /. f → fff &;
H := f@{e4, e6, e7, #} /. f → fff &;
```

In[2416]:=
```
{F@#, G@#, H@#} & /@ {e0}
f[Flatten@%] /. f → fff
f[Flatten@%%, True] /. f → fff
f[Flatten@%%%, both] /. f → fff
```

Out[2416]= $\begin{pmatrix} e_5 & e_3 & -e_1 \end{pmatrix}$

Out[2417]= e_4

Out[2418]= $-e_4$

Out[2419]= $-e_4$

In[2420]:=
```
(* Left to Right *)
((F@e0) // G) // H
```
Out[2420]= $-e_4$

In[2421]:=
```
(* Right to Left *)
F@(G@(H@e0))
```
Out[2421]= $-e_4$

Redefine Λ & g as functions: Λ_n for n = 1 – 8 g_n for n = 9 – 14 (note: similar structure with $\frac{i}{2}$ factor)

In[2422]:=
```
Clear[Λ, Λ1, Λ2, Λ3, Λ4, Λ5, Λ6, Λ7, Λ8, g]
```

In[2423]:=
```
Λ[n_, ff_] := (**)Total[(**)f@{#, ff} & /@ Λn(**)](**);
g[n_, ff_] := (**)Total[(**)f@{gn, ff} & /@ gn(**)](**);
```

In[2425]:=
```
(* Λ1→-e6/2 *)
```

In[2426]:=
```
Λ1 = i/2 {{e1, e5}, -{e3, e4}};
Λ1@ff_ := Λ[1, ff];
Λ1@e0 /. f → fff
```
Out[2428]= $-\frac{e_6}{2}$

In[2429]:=
```
(* Λ1f→ν] *)
```

In[2430]:=
```
Λ[1, ν] /. f → fff
```
Out[2430]= $\frac{1}{4} i (e_2 + i e_6)$

In[2431]:= `(* Λ2→±e2/2→0 *)`

In[2437]:= `(* Λ3→-e7/2 *)`

In[2443]:= `(* Λ4→±e3/2→0 *)`

In[2449]:= `(* Λ5→-e1/2 *)`

In[2455]:= `(* Λ6→±e5/2→0 *)`

In[2461]:= `(* Λ7→∓e4/2→0 *)`

In[2467]:= `(* Λ8→-e7/2 *)`

In[2473]:= `(* g9→-e6/12^2 *)`

In[2479]:= `(* g10→+e2/12^2 *)`

In[2485]:= `(* g11→+e3/12^2 *)`

In[2491]:= `(* g12→+e1/12^2 *)`

In[2497]:= `(* g13→+e5/24 *)`

In[2503]:= `(* g14→+e4/12^2 *)`

Three generation particle models

Redefine raising & lowering operators $\{\alpha_1, \alpha_2, \alpha_3\}$ as functions of n = 1 – 3 and f

Define SU (1) Q as a function of α_n n = 1 – 3 and f

In[2521]:=
```
Clear@su1Q;
```

In[2522]:=
```
su1Q := Sum[1/3 f@{αi†, αi, #}, {i, 1, 3}] &;
```

In[2523]:=
```
su1Q@e0 /. f → fff
%†
%%*
```

Out[2523]= $\frac{1}{2} i (e_7 + i)$

Out[2524]= $\frac{1}{2} - \frac{i e_7}{2}$

Out[2525]= $-\frac{1}{2} + \frac{i e_7}{2}$

In[2526]:=
```
su1Q@e4
Simplify@%
```

Out[2526]= $\frac{1}{3} f\left(\left\{-\frac{e_3}{2} - \frac{i e_1}{2}, \frac{1}{2}(-e_3 + i e_1), e_4\right\}\right) + \frac{1}{3} f\left(\left\{-\frac{e_5}{2} - \frac{i e_4}{2}, \frac{1}{2}(-e_5 + i e_4), e_4\right\}\right) + \frac{1}{3} f\left(\left\{-\frac{e_6}{2} - \frac{i e_2}{2}, \frac{1}{2}(-e_6 + i e_2), e_4\right\}\right)$

Out[2527]= $\frac{1}{3}\left(f\left(\left\{-\frac{1}{2} i (e_1 - i e_3), \frac{1}{2} i (e_1 + i e_3), e_4\right\}\right) + f\left(\left\{-\frac{1}{2} i (e_4 - i e_5), \frac{1}{2} i (e_4 + i e_5), e_4\right\}\right) + f\left(\left\{-\frac{1}{2} i (e_2 - i e_6), \frac{1}{2} i (e_2 + i e_6), e_4\right\}\right)\right)$

In[2528]:=
```
f@{α#†, α#} /. f → fff & /@ Range@3
```

Out[2528]= $\left\{\frac{1}{2} i (e_7 + i), \frac{1}{2} i (e_7 + i), \frac{1}{2} i (e_7 + i)\right\}$

Define exp G as a function of j = 1 – 8 → r_j and f

In[2529]:=
```
Clear@G
```

In[2530]:=
```
r0 = 2.;
r1 = r2 = r3 = r4 = r5 = r6 = r7 = r8 = 1.;
G[j_, ff_] := ⅇ^(ⅈ rj Λ[j,ff] + ⅈ r0 su1Q@ff);
```

In[2533]:=
```
ⅈ r1 Λ[1, 1] /. f → fff
ⅈ r0 su1Q@e0 /. f → fff
```

Out[2533]= $(0. - 0.5\, i)\, e_6$

Out[2534]= $(-1. + 0.\, i)(e_7 + i)$

In[2535]:=
```
G[1, 1] /. f → fff
```

Out[2535]= $\exp((0. - 0.5\, i)\, e_6 - (1. + 0.\, i)(e_7 + i))$

In[2536]:=
```
Chop@{#, G[1, 1] α# G[1, 1]^-1, G[1, 1] α#† G[1, 1]^-1} & /@ Range@3 /. f → fff
```

Out[2536]=
$$\begin{pmatrix} 1 & 0.5\,(i e_4 - e_5) & 1.\left(-\frac{i e_4}{2} - \frac{e_5}{2}\right) \\ 2 & 0.5\,(i e_1 - e_3) & 1.\left(-\frac{i e_1}{2} - \frac{e_3}{2}\right) \\ 3 & 0.5\,(i e_2 - e_6) & 1.\left(-\frac{i e_2}{2} - \frac{e_6}{2}\right) \end{pmatrix}$$

Define the 3 generations of quarks and leptons (2014 and 2019 versions)

In[2537]:=
```
Clear[u1, u2, u3, Λ1Λ2, Λ4Λ5, Λ6Λ7,
 u1r, u1g, u1b, u2r, u2g, u2b, u3r, u3g, u3b,
 d123rgb, d1r, d1g, d1b, d2r, d2g, d2b, d3r, d3g, d3b,
 ν1, ν2, ν3, e123, e1, e2, e3]
```

In[2538]:=
```
u1@oct_ := {sS[[2]], Flatten@{Total@oct}, sS[[4]]};
u2@oct_ := {sS[[3]], Flatten@{Total@oct}, sS[[1]]};
u3@oct_ := {sS[[4]], Flatten@{Total@oct}, sS[[1]]};
```

In[2541]:=
```
Λnm[nm_List] := f@# & /@ Flatten[{nm}, 1];
```

In[2542]:=
```
(* u1r construction / deconstruction *)
```

In[2543]:=
```
-ⅈ Λ6 {-1, 1} // StandardForm
Λnm[-ⅈ Λ6] {-1, 1}
Total@% /. f → fff
```

Out[2543]//StandardForm=
$$\left\{\left\{-\frac{e_1}{2}, -\frac{e_6}{2}\right\}, \left\{\frac{e_2}{2}, \frac{e_3}{2}\right\}\right\}$$

Out[2544]= $\left\{-f\left(\left\{\frac{e_1}{2}, \frac{e_6}{2}\right\}\right), f\left(\left\{\frac{e_2}{2}, \frac{e_3}{2}\right\}\right)\right\}$

Out[2545]= $\frac{e_5}{2}$

In[2546]:=
```
Λ7 {-1, 1} // StandardForm
Λnm[Λ7] {-1, 1}
Total@% /. f → fff
```

Out[2546]//StandardForm=
$$\left\{\left\{-\frac{i e_1}{2}, -\frac{i e_2}{2}\right\}, \left\{\frac{i e_3}{2}, \frac{i e_6}{2}\right\}\right\}$$

Out[2547]= $\left\{-f\left(\left\{\frac{i e_1}{2}, \frac{i e_2}{2}\right\}\right), f\left(\left\{\frac{i e_3}{2}, \frac{i e_6}{2}\right\}\right)\right\}$

Out[2548]= $\frac{e_4}{2}$

In[2549]:=
```
Λ6Λ7 = (Λnm[-ⅈ Λ6] + Λnm[Λ7]) {-1, 1}
% /. f → fff
Total@%
```

Out[2549]= $\left\{-f\left(\left\{\frac{e_1}{2}, \frac{e_6}{2}\right\}\right) - f\left(\left\{\frac{i e_1}{2}, \frac{i e_2}{2}\right\}\right), f\left(\left\{\frac{e_2}{2}, \frac{e_3}{2}\right\}\right) + f\left(\left\{\frac{i e_3}{2}, \frac{i e_6}{2}\right\}\right)\right\}$

Out[2550]= $\left\{\frac{e_4}{4} + \frac{e_5}{4}, \frac{e_4}{4} + \frac{e_5}{4}\right\}$

Out[2551]= $\frac{e_4}{2} + \frac{e_5}{2}$

Use this change for 2014 u1 R→G fix:

In[2552]:=
```
Λ6Λ7 = (**)-2(*)1**) (ⅈ Λnm[Λ6] + Λnm[Λ7]) {-1, 1}
Expand@% /. f → fff
Total@%
```

Out[2552]= $\left\{2\left(f\left(\left\{\frac{i e_1}{2}, \frac{i e_2}{2}\right\}\right) + i f\left(\left\{\frac{i e_1}{2}, \frac{i e_6}{2}\right\}\right)\right), -2\left(i f\left(\left\{\frac{i e_2}{2}, \frac{i e_3}{2}\right\}\right) + f\left(\left\{\frac{i e_3}{2}, \frac{i e_6}{2}\right\}\right)\right)\right\}$

Out[2553]= $\left\{-\frac{e_4}{2} + \frac{i e_5}{2}, -\frac{e_4}{2} + \frac{i e_5}{2}\right\}$

Out[2554]= $-e_4 + i e_5$

In[2555]:=
```
(* u1g construction / deconstruction *)
```

In[2556]:=
```
ⅈ Λ4 {1, -1} // StandardForm
Λnm[ⅈ Λ4] {1, -1}
Total@% /. f → fff
```

Out[2556]//StandardForm=
$$\left\{\left\{-\frac{e_2}{2}, -\frac{e_5}{2}\right\}, \left\{\frac{e_4}{2}, \frac{e_6}{2}\right\}\right\}$$

Out[2557]= $\left\{f\left(\left\{-\frac{e_2}{2}, -\frac{e_5}{2}\right\}\right), -f\left(\left\{-\frac{e_4}{2}, -\frac{e_6}{2}\right\}\right)\right\}$

Out[2558]= $-\frac{e_3}{2}$

In[2559]:=
```
Λ5 {1, -1} // StandardForm
Λnm[Λ5] {1, -1}
Total@% /. f → fff
```

Out[2559]//StandardForm=
$$\left\{\left\{-\frac{i\, e_5}{2}, -\frac{i\, e_6}{2}\right\}, \left\{-\frac{i\, e_2}{2}, -\frac{i\, e_4}{2}\right\}\right\}$$

Out[2560]= $\left\{f\left(\left\{-\frac{i\, e_5}{2}, -\frac{i\, e_6}{2}\right\}\right), -f\left(\left\{\frac{i\, e_2}{2}, \frac{i\, e_4}{2}\right\}\right)\right\}$

Out[2561]= 0

In[2562]:=
```
Λ4Λ5 = (Λnm[i Λ4] + Λnm[Λ5]) {1, -1}
% /. f → fff
Total@%
```

Out[2562]= $\left\{f\left(\left\{-\frac{e_2}{2}, -\frac{e_5}{2}\right\}\right) + f\left(\left\{-\frac{i\, e_5}{2}, -\frac{i\, e_6}{2}\right\}\right), -f\left(\left\{-\frac{e_4}{2}, -\frac{e_6}{2}\right\}\right) - f\left(\left\{\frac{i\, e_2}{2}, \frac{i\, e_4}{2}\right\}\right)\right\}$

Out[2563]= $\left\{-\frac{e_1}{4} - \frac{e_3}{4}, \frac{e_1}{4} - \frac{e_3}{4}\right\}$

Out[2564]= $-\frac{e_3}{2}$

Use this change for 2014 u1 G→R fix:

In[2565]:=
```
Λ4Λ5 = (**)2(*)4**) (Λnm[Λ4] {1, -1} + i Λnm[Λ5])
Expand[% /. f → fff]
Total@%
```

Out[2565]= $\left\{2\left(f\left(\left\{\frac{i\, e_2}{2}, \frac{i\, e_5}{2}\right\}\right) + i\, f\left(\left\{-\frac{i\, e_5}{2}, -\frac{i\, e_6}{2}\right\}\right)\right), 2\left(i\, f\left(\left\{\frac{i\, e_2}{2}, \frac{i\, e_4}{2}\right\}\right) - f\left(\left\{\frac{i\, e_4}{2}, \frac{i\, e_6}{2}\right\}\right)\right)\right\}$

Out[2566]= $\left\{\frac{e_3}{2} - \frac{i\, e_1}{2}, \frac{e_3}{2} - \frac{i\, e_1}{2}\right\}$

Out[2567]= $e_3 - i\, e_1$

In[2568]:=
```
(* u1b construction / deconstruction *)
```

In[2569]:=
```
i Λ1 {-1, 1} // StandardForm
Λnm[i Λ1] {-1, 1}
Total@% /. f → fff
```

Out[2569]//StandardForm=
$$\left\{\left\{\frac{e_1}{2}, \frac{e_5}{2}\right\}, \left\{\frac{e_3}{2}, \frac{e_4}{2}\right\}\right\}$$

Out[2570]= $\left\{-f\left(\left\{-\frac{e_1}{2}, -\frac{e_5}{2}\right\}\right), f\left(\left\{\frac{e_3}{2}, \frac{e_4}{2}\right\}\right)\right\}$

Out[2571]= 0

In[2572]:=
```
Λ2 {-1, 1} // StandardForm
Λnm[Λ2] {-1, 1}
Total@% /. f → fff
```

Out[2572]//StandardForm=
$$\left\{\left\{\frac{i\, e_1}{2}, \frac{i\, e_4}{2}\right\}, \left\{-\frac{i\, e_3}{2}, -\frac{i\, e_5}{2}\right\}\right\}$$

Out[2573]= $\left\{-f\left(\left\{-\frac{i\, e_1}{2}, -\frac{i\, e_4}{2}\right\}\right), f\left(\left\{-\frac{i\, e_3}{2}, -\frac{i\, e_5}{2}\right\}\right)\right\}$

Out[2574]= $-\frac{e_2}{2}$

In[2575]:=
```
Λ1Λ2 = (Λnm[i Λ1] + Λnm[Λ2]) {-1, 1}
% /. f → fff
Total@%
```

Out[2575]= $\left\{-f\left(\left\{-\frac{e_1}{2}, -\frac{e_5}{2}\right\}\right) - f\left(\left\{-\frac{i\,e_1}{2}, -\frac{i\,e_4}{2}\right\}\right), f\left(\left\{\frac{e_3}{2}, \frac{e_4}{2}\right\}\right) + f\left(\left\{-\frac{i\,e_3}{2}, -\frac{i\,e_5}{2}\right\}\right)\right\}$

Out[2576]= $\left\{-\frac{e_2}{4} - \frac{e_6}{4}, \frac{e_6}{4} - \frac{e_2}{4}\right\}$

Out[2577]= $-\frac{e_2}{2}$

In[2578]:=
```
(* u1 RGB *)
```

In[2579]:=
```
u1r := u1[Λ6Λ7];
u1r
u1r /. f → fff
```

Out[2580]= $\left\{\{s, S^*\}, \left\{2\left(f\left(\left\{\frac{i\,e_1}{2}, \frac{i\,e_2}{2}\right\}\right) + i\,f\left(\left\{\frac{i\,e_1}{2}, \frac{i\,e_6}{2}\right\}\right)\right) - 2\left(i\,f\left(\left\{\frac{i\,e_2}{2}, \frac{i\,e_3}{2}\right\}\right) + f\left(\left\{\frac{i\,e_3}{2}, \frac{i\,e_6}{2}\right\}\right)\right)\right\}, \{s, S\}\right\}$

Out[2581]= $\left\{\{s, S^*\}, \left\{2\left(-\frac{e_4}{4} + \frac{i\,e_5}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i\,e_5}{4}\right)\right\}, \{s, S\}\right\}$

In[2582]:=
```
(* From 2014 paper *)
%[[2, 1]] ∘ v
```

Out[2582]= $-e_4 + i\,e_5$

In[2583]:=
```
u1g := u1[Λ4Λ5];
u1g
u1g /. f → fff
```

Out[2584]= $\left\{\{s, S^*\}, \left\{2\left(i\,f\left(\left\{\frac{i\,e_2}{2}, \frac{i\,e_4}{2}\right\}\right) - f\left(\left\{\frac{i\,e_4}{2}, \frac{i\,e_6}{2}\right\}\right)\right) + 2\left(f\left(\left\{\frac{i\,e_2}{2}, \frac{i\,e_5}{2}\right\}\right) + i\,f\left(\left\{-\frac{i\,e_5}{2}, -\frac{i\,e_6}{2}\right\}\right)\right)\right\}, \{s, S\}\right\}$

Out[2585]= $\left\{\{s, S^*\}, \left\{4\left(\frac{e_3}{4} - \frac{i\,e_1}{4}\right)\right\}, \{s, S\}\right\}$

In[2586]:=
```
(* From 2014 paper *)
%[[2, 1]] ∘ v
```

Out[2586]= 0

In[2587]:=
```
u1b := u1[Λ1Λ2];
u1b
u1b /. f → fff
```

Out[2588]= $\left\{\{s, S^*\}, \left\{-f\left(\left\{-\frac{e_1}{2}, -\frac{e_5}{2}\right\}\right) - f\left(\left\{-\frac{i\,e_1}{2}, -\frac{i\,e_4}{2}\right\}\right) + f\left(\left\{-\frac{i\,e_3}{2}, -\frac{i\,e_5}{2}\right\}\right) + f\left(\left\{\frac{e_3}{2}, \frac{e_4}{2}\right\}\right)\right\}, \{s, S\}\right\}$

Out[2589]= $\left\{\{s, S^*\}, \left\{-\frac{e_2}{2}\right\}, \{s, S\}\right\}$

In[2590]:=
```
(* From 2014 paper *)
%[[2, 1]] ∘ v
```

Out[2590]= $-\frac{e_2}{4} + \frac{i\,e_6}{4}$

In[2591]:=
```
(* u2 RGB *)
```

In[2604]:=
```
(* u3 RGB *)
```

In[2624]:=
```
(* d1 RGB *)
```

In[2625]:=
```
d123rgb@oct_ := (*)Row@**){sS[[2]], Total@oct, sS[[1]]};
```

In[2626]:=
```
d1r := d123rgb[{{ f@{e1, e2, e6} - f@{e3}},        - {f@{e1, e4, e5} + i f@{e1}}}];
d1r
d1r /. f → fff
```

Out[2627]= $\{\{s, S^*\}, \{f(\{e_1, e_2, e_6\}) - f(\{e_1, e_4, e_5\}) - i\, f(\{e_1\}) - f(\{e_3\})\}, \{s^*, S\}\}$

Out[2628]= $\{\{s, S^*\}, \{-e_3 - i\, e_1\}, \{s^*, S\}\}$

In[2629]:=
```
(* From 2014 paper *)
%[[2, 1]] ∘ ν
```

Out[2629]= $-e_3 - i\, e_1$

In[2630]:=
```
d1g := d123rgb[{{ f@{e1, e3, e4} + f@{e5}},        i {-f@{e1, e3, e5} + f@{e4}}}];
(* ~u3r *)
d1g
d1g /. f → fff
```

Out[2631]= $\{\{s, S^*\}, \{f(\{e_1, e_3, e_4\}) + i\,(f(\{e_4\}) - f(\{e_1, e_3, e_5\})) + f(\{e_5\})\}, \{s^*, S\}\}$

Out[2632]= $\{\{s, S^*\}, \{0\}, \{s^*, S\}\}$

In[2633]:=
```
(* From 2014 paper *)
%[[2, 1]] ∘ ν
```

Out[2633]= 0

In[2634]:=
```
d1b := d123rgb[{-{f@{e1, e2, e5} + f@{e1, e4, e6}},  i {f@{e1, e5, e6} - f@{e1, e2, e4}}}];
d1b
d1b /. f → fff
```

Out[2635]= $\{\{s, S^*\}, \{-f(\{e_1, e_2, e_5\}) - f(\{e_1, e_4, e_6\}) + i\,(f(\{e_1, e_5, e_6\}) - f(\{e_1, e_2, e_4\}))\}, \{s^*, S\}\}$

Out[2636]= $\{\{s, S^*\}, \{0\}, \{s^*, S\}\}$

In[2637]:=
```
(* From 2014 paper *)
%[[2, 1]] ∘ ν
```

Out[2637]= 0

In[2638]:=
```
(* d2 RGB *)
```

In[2651]:=
```
(* d3 RGB *)
```

In[2664]:=
```
(* ν *)
```

In[2665]:=
```
ν1 := (*)Row@**){sS[[1]],  Total@ {{1},  i {f@{e1, e3} + f@{e2, e6} + f@{e4, e5}}}, sS[[1]]};
ν1 /. f → fff
```

Out[2666]= $\{\{s^*, S\}, \{1 + 3\, i\, e_7\}, \{s^*, S\}\}$

In[2667]:=
```
(* From 2014 paper *)
ν1 := (*)Row@**)
  {sS[[1]],         {1 + i 0.},                                sS[[1]]};
ν1 /. f → fff
%[[2, 1]] ∘ ν
```

Out[2668]= $\{\{s^*, S\}, \{1. + 0.\, i\}, \{s^*, S\}\}$

Out[2669]= $0.5 + (0. + 0.5\, i)\, e_7$

In[2670]:=
```
(*                                                              ~
 ν1 *)
ν2 := (*)Row@**){sS[[2]],  Total@{{3}, -i {f@{e1, e3} + f@{e2, e6} + f@{e4, e5}}}, sS[[2]]};
ν2 /. f → fff
```

Out[2671]= $\{\{s, S^*\}, \{3 - 3\, i\, e_7\}, \{s, S^*\}\}$

In[2672]:=
```
(* From 2014 paper *)
(*
 ν1 *)
ν2 := (*)Row@**){sS[[2]],   Total@{{f@{e1, e3} + f@{e2, e6} + f@{e4, e5}}},          sS[[2]]};
ν2 /. f → fff
%[[2, 1]] ∘ ν
```

Out[2673]= $\{\{\text{s}, S^*\}, \{3\, e_7\}, \{\text{s}, S^*\}\}$

Out[2674]= $\frac{3\, e_7}{2} - \frac{3\, i}{2}$

In[2675]:=
```
(*
 d1b *)
ν3 := (*)Row@**){sS[[3]],
   Total@{-{f@{e1, e2, e5} + f@{e1, e4, e6}}, - i {f@{e1, e5, e6} + f@{e1, e2, e4}}}, sS[[4]]};
ν3 /.
 f →
  fff
```

Out[2676]= $\{\{s^*, S^*\}, \{2\, i\}, \{\text{s}, \text{S}\}\}$

In[2677]:=
```
(* From 2014 paper *)
%[[2, 1]] ∘ ν
```

Out[2677]= $-e_7 + i$

In[2678]:=
```
(* ν *)
```

Define SU(3) Q to match the action in eqs. 19, 22 and 23

While I've tried many variations, this one is simply based on the position in the list of sS conjugate combinations (given the leading sS pair in the defined particle.

In[2695]:=
```
su3Q@n_ := Module[{in = n[[2]],
    pos = position[sS, n[[1]]]},
   (**)Simplify@(**)If[pos === {}, in, {(**)i, (**) pos/3, n[[1, 1]]}(*)/.
     {s→1/2 (f@{in}+i/2(-f@{e7,in}+f@{e1, e3,in}+f@{e2, e6,in}+f@{e4, e5,in})),
      S→1/2 (f@{in}+i/2(-f[{e7,in},True]+
             f[{e1, e3,in},True]+f[{e2, e6,in},True]+f[{e4, e5,in},True]))}**)]];
```

This works for the example given in u1r,

but I can't figure out how to make it work across all flavors and generations given their particle definitions above. My confusion seems to be around the proper use of ther sS components.

Do an example, showing the decomposition of sS and show su3Q taking u1r as input (with and without evaluating f):

In[2696]:=
```
doConjugate = True;
```

In[2697]:=
```
sS /. {s → 1/2 (f@{in} + i/2 (-f@{e7, in} + f@{e1, e3, in} + f@{e2, e6, in} + f@{e4, e5, in})),
   S → 1/2 (f@{in} + i/2 (-f[{e7, in}, True] + f[{e1, e3, in}, True] +
          f[{e2, e6, in}, True] + f[{e4, e5, in}, True]))} /. {in →  1, f → fff}
```

Out[2697]=
$$\begin{pmatrix} \frac{1}{2}(1 - i\, e_7) & \frac{1}{2}(i\, e_7 + 1) \\ \frac{1}{2}(i\, e_7 + 1) & \frac{1}{2}(1 - i\, e_7) \\ \frac{1}{2}(1 - i\, e_7) & \frac{1}{2}(1 - i\, e_7) \\ \frac{1}{2}(i\, e_7 + 1) & \frac{1}{2}(i\, e_7 + 1) \end{pmatrix}$$

In[2698]:=
```
Row@u1r
```

Out[2698]= $\{s, S^*\}\left\{2\left(f\left(\left\{\frac{i\,e_1}{2}, \frac{i\,e_2}{2}\right\}\right) + i\,f\left(\left\{\frac{i\,e_1}{2}, \frac{i\,e_6}{2}\right\}\right)\right) - 2\left(i\,f\left(\left\{\frac{i\,e_2}{2}, \frac{i\,e_3}{2}\right\}\right) + f\left(\left\{\frac{i\,e_3}{2}, \frac{i\,e_6}{2}\right\}\right)\right)\right\}\{s, S\}$

In[2699]:=
```
u1r[[2, 1]] /. f → fff
```

Out[2699]= $2\left(-\frac{e_4}{4} + \frac{i\,e_5}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i\,e_5}{4}\right)$

In[2700]:=
```
Expand@ #/3 {f@{sS[[#, 1]]}, f@{sS[[#, 2]]}} /.
    { s → 1/2 (f@{in} + i/2 (-f@{e7, in} + f@{e1, e3, in} + f@{e2, e6, in} + f@{e4, e5, in})),
     S → 1/2 (f@{in} + i/2 (-f[{e7, in}, True] + f[{e1, e3, in}, True] +
            f[{e2, e6, in}, True] + f[{e4, e5, in}, True]))} /.
   {in → u1r[[2, 1]] /. f → fff} & /@ Range@3
% /. f → fff
Expand@Total[Total /@ %]
```

Out[2700]= $\begin{pmatrix} \frac{1}{3} f\left(\left\{\frac{1}{2}\left(f\left(\left\{2\left(\frac{i e_5}{4} - \frac{e_4}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i e_5}{4}\right)\right\}\right) + \frac{1}{2} i\left(-f\left(\left\{e_7, 2\left(\frac{i e_5}{4} - \frac{e_4}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i e_5}{4}\right)\right\}\right) + f\left(\left\{e_1, e_3, 2\left(\frac{i e_5}{4} - \frac{e_4}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i e_5}{4}\right)\right\}\right) + f\left(\left\{e_2, e_6, 2\left(\frac{i e_5}{4} - \right.\right.\right.\right.\right.\right. \\ \frac{2}{3} f\left(\left\{\frac{1}{2}\left(f\left(\left\{2\left(\frac{i e_5}{4} - \frac{e_4}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i e_5}{4}\right)\right\}\right) + \frac{1}{2} i\left(-f\left(\left\{e_7, 2\left(\frac{i e_5}{4} - \frac{e_4}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i e_5}{4}\right)\right\}\right) + f\left(\left\{e_1, e_3, 2\left(\frac{i e_5}{4} - \frac{e_4}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i e_5}{4}\right)\right\}\right) + f\left(\left\{e_2, e_6, 2\left(\frac{i e_5}{4} - \right.\right.\right.\right.\right.\right. \\ f\left(\left\{\frac{1}{2}\left(f\left(\left\{2\left(\frac{i e_5}{4} - \frac{e_4}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i e_5}{4}\right)\right\}\right) + \frac{1}{2} i\left(-f\left(\left\{e_7, 2\left(\frac{i e_5}{4} - \frac{e_4}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i e_5}{4}\right)\right\}\right) + f\left(\left\{e_1, e_3, 2\left(\frac{i e_5}{4} - \frac{e_4}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i e_5}{4}\right)\right\}\right) + f\left(\left\{e_2, e_6, 2\left(\frac{i e_5}{4} - \right.\right.\right.\right.\right.\right. \end{pmatrix}$

Out[2701]= $\begin{pmatrix} \frac{1}{3}(-e_4 - i\,e_5) & 0 \\ \frac{2}{3}(i\,e_5 - e_4) & 0 \\ -e_4 - i\,e_5 & 0 \end{pmatrix}$

Out[2702]= $-2\,e_4 - \frac{2\,i\,e_5}{3}$

In[2703]:=
```
doConjugate = False;
su3Q@u1r[[2, 1]]
```

Out[2704]= $-2\left(i\,f\left(\left\{\frac{i\,e_2}{2}, \frac{i\,e_3}{2}\right\}\right) + f\left(\left\{\frac{i\,e_3}{2}, \frac{i\,e_6}{2}\right\}\right)\right)$

In[2705]:=
```
doConjugate = True;
su3Q@u1r[[2, 1]]
```

Out[2706]= $-2\left(i\,f\left(\left\{\frac{i\,e_2}{2}, \frac{i\,e_3}{2}\right\}\right) + f\left(\left\{\frac{i\,e_3}{2}, \frac{i\,e_6}{2}\right\}\right)\right)$

From the 2014 particle model in [2] show commutation action for U1R→U1G with octonion multiplication

Cohl doesn't explicitly reduce the octonion elements via octonionic multiplication here, but does reduce them via common identities. Here we process the manual reduction of the color changing action of $\Lambda_1[\nu] \odot$ u1r $== \nu \circ$ u1g shown in eqs. 20 & 21 of [2], and then show the commutation directly.

So here we replicate the identity reductions, but also process each step with octonionic multiplication without the $\circ$ v:

In[2707]:=
```
{-i f@{e1, e5, e1, e2}, -f@{e1, e5, e1, e6}, +f@{e1, e5, e2, e3}, +i f@{e1, e5, e3, e6}} /. f → fff
Total@%
```

Out[2707]= $\{-i\,e_3, -e_1, -e_1, -i\,e_3\}$

Out[2708]= $-2\,e_1 - 2\,i\,e_3$

In[2709]:=
```
{+ⅈ f@{e3, e4, e1, e2}, +f@{e3, e4, e1, e6}, -f@{e3, e4, e2, e3}, -ⅈ f@{e3, e4, e3, e6}} /. f → fff
Total@%
```

Out[2709]= $\{-i\, e_3, -e_1, -e_1, -i\, e_3\}$

Out[2710]= $-2\, e_1 - 2\, i\, e_3$

In[2711]:=
```
{+ⅈ f@{e1, e2, e1, e5}, +f@{e1, e6, e1, e5}, -f@{e2, e3, e1, e5}, -ⅈ f@{e3, e6, e1, e5}} /. f → fff
Total@%
```

Out[2711]= $\{-i\, e_3, -e_1, e_1, i\, e_3\}$

Out[2712]= 0

In[2713]:=
```
{-ⅈ f@{e1, e2, e3, e4}, -f@{e1, e6, e3, e4}, +f@{e2, e3, e3, e4}, +ⅈ f@{e3, e6, e3, e4}} /. f → fff
Total@%
```

Out[2713]= $\{i\, e_3, e_1, -e_1, -i\, e_3\}$

Out[2714]= 0

In[2715]:=
```
(* Last reduction in eq. 20 *)
Expand@Total[ⅈ/2 {%%%%%%%, %%%%%, %%%, %}]
```

Out[2715]= $2\, e_3 - 2\, i\, e_1$

In[2716]:=
```
(* Last reduction in eq. 21 *)
ⅈ {ⅈ f@{e2, e5}, -f@{e5, e6}, -f@{e2, e4}, -ⅈ f@{e4, e6}} /. f → fff
Total@%
```

Out[2716]= $\{e_3, -i\, e_1, -i\, e_1, e_3\}$

Out[2717]= $2\, e_3 - 2\, i\, e_1$

The above reduction did not apply the final ∘*v*. If we do that drives the result to zero.

In[2718]:=
```
% ∘ v
```

Out[2718]= 0

Now we perform the commutation action in one step (Λ1[$-\nu^*$]⊙u1r⟦2,1⟧, notice the use of $-\nu^*$ not *v*, which would drive the result to zero).
There is also a factors of 2 that is related to the change in basis from 2014 to 2019. Then compare it against u1g and the result from the manual decomposition above:

In[2719]:=
```
(**)
Expand[2 (Λ[1, -ν*] /. f → fff) ⊙ (u1r⟦2, 1⟧ /. f → fff)]
Expand[2 u1g⟦2, 1⟧ /. f → fff]
Expand[% == %% == %%%%]
```

Out[2719]= $2\, e_3 - 2\, i\, e_1$

Out[2720]= $2\, e_3 - 2\, i\, e_1$

Out[2721]= True

Now test the reverse action (Λ1[ν]⊙u1g⟦2,1⟧, for which we need to use *v* (not $-\nu^*$ as above) which would drive this result to zero):

In[2722]:=
```
(* u1 G→R *)
(Λ[1, ν] /. f → fff) ⊙ (u1g⟦2, 1⟧ /. f → fff)
u1r⟦2, 1⟧ /. f → fff
Expand[% == %%]
```

Out[2722]= $-e_4 + i\, e_5$

Out[2723]= $2\left(-\frac{e_4}{4} + \frac{i\, e_5}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i\, e_5}{4}\right)$

Out[2724]= True

Moving on, we now build the machinery to perform these commutations across the SM particles:

In[2725]:=
```
(* Function to output the action with input values of Λnν and the bioctonion particle *)
doAction[Λg_, particle_] := Expand[(Λg /. f → fff) ⊙ (particle /. f → fff)];
```

In[2726]:=
```
(* u1 R→G *)
```

In[2727]:=
```
doAction[Λ[1, -ν*], u1r⟦2, 1⟧]
u1g⟦2, 1⟧ /. f → fff
Expand[% == %%]
```

Out[2727]= $e_3 - i\, e_1$

Out[2728]= $4\left(\frac{e_3}{4} - \frac{i\, e_1}{4}\right)$

Out[2729]= True

In[2730]:=
```
(* u1 G→R *)
```

In[2731]:=
```
doAction[Λ1[ ν], u1g⟦2, 1⟧]
u1r⟦2, 1⟧ /. f → fff
Expand[% == %%]
```

Out[2731]= $-e_4 + i\, e_5$

Out[2732]= $2\left(-\frac{e_4}{4} + \frac{i\, e_5}{4}\right) - 2\left(\frac{e_4}{4} - \frac{i\, e_5}{4}\right)$

Out[2733]= True

Function to output the action across SM particles with a string input identifying Λ_n.

I need more information on the flavor and color transformations involved in Λ_n, but this helps deduce that by trying various combinations.

In[2734]:=
```
(* Define the list of particle flavors *)
flavorList = Flatten@pTypeList⟦ ;; 2⟧
```

Out[2734]= {e, ν, d, u}

In[2735]:=
```
doSMaction@in_ := Column[Table[Column[Table[Row[Table[
        inExp = ToExpression@in;
        prt = ToExpression[flavorList⟦flv⟧ <> ToString@gen <> If[flv > 2, rgb⟦col⟧, " "]];
        sprt = ToString[
          Subsuperscript[flavorList⟦flv⟧, gen, If[flv > 2, rgb⟦col⟧, " "]], TraditionalForm];
        ssprt = Style[sprt, Bold, Red];
        (* skip color iterations on leptons *)
        If[flv < 3 && col > 1, Nothing,
         Column[{ssprt, Row@{sprt <> "=    ", Row@{Expand@prt⟦2, 1⟧ /. f → fff}, "      "},
             Row@{in <> "⊙" <> sprt <> "=    ",
             doAction[inExp, prt⟦2, 1⟧ /. f → fff], "      "}}, Center]],
        {col, 3}]],
      {flv, 4}], Center],
    {gen, 3}], Center];
```

In[2736]:=
```
(* u1 R→G *)
doSMaction@"Λ[1,-ν*]"
```

Out[2736]=

e_1

$e_1 = -3\, e_3 - i\, e_1$

$\Lambda[1,-\nu^*]\odot e_1 = -2\, e_4 - 2\, i\, e_5$

ν_1

$\nu_1 = 1. + 0.\, i$

$\Lambda[1,-\nu^*]\odot \nu_1 = 0$

d_1^r	d_1^g	d_1^b
$d_1^r = -e_3 - i\, e_1$	$d_1^g = 0$	$d_1^b = 0$
$\Lambda[1,-\nu^*]\odot d_1^r = -e_4 - i\, e_5$	$\Lambda[1,-\nu^*]\odot d_1^g = 0$	$\Lambda[1,-\nu^*]\odot d_1^b = 0$

u_1^r	u_1^g	u_1^b
$u_1^r = -e_4 + i\, e_5$	$u_1^g = e_3 - i\, e_1$	$u_1^b = -\frac{e_2}{2}$
$\Lambda[1,-\nu^*]\odot u_1^r = e_3 - i\, e_1$	$\Lambda[1,-\nu^*]\odot u_1^g = 0$	$\Lambda[1,-\nu^*]\odot u_1^b = \frac{e_7}{4}$

e_2

$e_2 = 2\, e_6 + 2\, i\, e_2$

$\Lambda[1,-\nu^*]\odot e_2 = 0$

ν_2

$\nu_2 = 3\, e_7$

$\Lambda[1,-\nu^*]\odot \nu_2 = \frac{3\, e_2}{2} - \frac{3\, i\, e_6}{2}$

d_2^r	d_2^g	d_2^b
$d_2^r = 2\, e_6 + 2\, i\, e_2$	$d_2^g = 2\, e_7 + 2\, i$	$d_2^b = -2\, e_5 - 2\, i\, e_4$
$\Lambda[1,-\nu^*]\odot d_2^r = 0$	$\Lambda[1,-\nu^*]\odot d_2^g = e_2 - i\, e_6$	$\Lambda[1,-\nu^*]\odot d_2^b = 2\, e_1 + 2\, i\, e_3$

u_2^r	u_2^g	u_2^b
$u_2^r = -e_4 + i\, e_5$	$u_2^g = e_3 - i\, e_1$	$u_2^b = -\frac{e_2}{2}$
$\Lambda[1,-\nu^*]\odot u_2^r = e_3 - i\, e_1$	$\Lambda[1,-\nu^*]\odot u_2^g = 0$	$\Lambda[1,-\nu^*]\odot u_2^b = \frac{e_7}{4}$

e_3

$e_3 = 2\, e_5 + 2\, i\, e_4$

$\Lambda[1,-\nu^*]\odot e_3 = -2\, e_1 - 2\, i\, e_3$

ν_3

$\nu_3 = 2\, i$

$\Lambda[1,-\nu^*]\odot \nu_3 = 0$

d_3^r	d_3^g	d_3^b
$d_3^r = 0$	$d_3^g = 0$	$d_3^b = e_3 + i\, e_1$
$\Lambda[1,-\nu^*]\odot d_3^r = 0$	$\Lambda[1,-\nu^*]\odot d_3^g = 0$	$\Lambda[1,-\nu^*]\odot d_3^b = e_4 + i\, e_5$

u_3^r	u_3^g	u_3^b
$u_3^r = 0$	$u_3^g = -e_3 - i\, e_1$	$u_3^b = 0$
$\Lambda[1,-\nu^*]\odot u_3^r = 0$	$\Lambda[1,-\nu^*]\odot u_3^g = -e_4 - i\, e_5$	$\Lambda[1,-\nu^*]\odot u_3^b = 0$

Visualize the entire family of flavors across 3 generations (2014 version)

These show the unevaluated form as well as the octonion reduced form, with the unevaluated (string form) showing the appending of “∘*v*” along with the evaluation of that:

In[2737]:=
```
doSM14 := Column[Table[Column[Table[Row[Table[
        prt = Expand[ToExpression[flavorList⟦flv⟧ <> ToString@gen <> If[flv > 2, rgb⟦col⟧, " "]]];
        sprt = ToString[
          Subsuperscript[flavorList⟦flv⟧, gen, If[flv > 2, rgb⟦col⟧, " "]], TraditionalForm];
        ssprt = Style[sprt, Bold, Red];
        sprt1 = ToString[Expand[prt⟦2, 1⟧], TraditionalForm] <> "     ";
        sprt2 = ToString[Expand[prt⟦2, 1⟧ /. f → fff], TraditionalForm];
        sprt3 = sprt <> "∘ν=" <> ToString[Expand[(prt⟦2, 1⟧ /. f → fff) ∘ ν], TraditionalForm];
        (* skip color iterations on leptons *)
        If[flv < 3 && col > 1, Nothing,
         Column[{ssprt, sprt1, sprt2, sprt3}, Center]],
        {col, 3}]],
      {flv, 4}], Center],
    {gen, 3}], Center];
```

In[2738]:=
```
doSM14
```

Out[2738]=

$\mathbf{e_1}$
$f(\{e_1,e_2,e_6\})+f(\{e_1,e_4,e_5\})-i f(\{e_1\})-f(\{e_3\})$
$-3e_3-ie_1$
$e_1\circ\nu=-2e_3-2ie_1$

$\boldsymbol{\nu_1}$
$1.+0.i$
$1.+0.i$
$\nu_1\circ\nu=0.5+(0.+0.5i)e_7$

$\mathrm{d}_1^{\mathfrak{r}}$
$f(\{e_1,e_2,e_6\})-f(\{e_1,e_4,e_5\})-i f(\{e_1\})-f(\{e_3\})$
$-e_3-ie_1$
$\mathrm{d}_1^{\mathfrak{r}}\circ\nu=-e_3-ie_1$

$\mathrm{d}_1^{\mathfrak{g}}$
$f(\{e_1,e_3,e_4\})-i f(\{e_1,e_3,e_5\})+i f(\{e_4\})+f(\{e_5\})$
0
$\mathrm{d}_1^{\mathfrak{g}}\circ\nu=0$

$\mathrm{d}_1^{\mathfrak{b}}$
$-i f(\{e_1,e_2,e_4\})-f(\{e_1,e_2,e_5\})-f(\{e_1,e_4,e_6\})+i f(\{e_1,e_5,e_6\})$
0
$\mathrm{d}_1^{\mathfrak{b}}\circ\nu=0$

$\mathrm{u}_1^{\mathfrak{r}}$
$2f\left(\left\{\frac{ie_1}{2},\frac{ie_2}{2}\right\}\right)+2i f\left(\left\{\frac{ie_1}{2},\frac{ie_6}{2}\right\}\right)-2i f\left(\left\{\frac{ie_2}{2},\frac{ie_3}{2}\right\}\right)-2f\left(\left\{\frac{ie_3}{2},\frac{ie_6}{2}\right\}\right)$
$-e_4+ie_5$
$\mathrm{u}_1^{\mathfrak{r}}\circ\nu=-e_4+ie_5$

$\mathrm{u}_1^{\mathfrak{g}}$
$2i f\left(\left\{\frac{ie_2}{2},\frac{ie_4}{2}\right\}\right)+2f\left(\left\{\frac{ie_2}{2},\frac{ie_5}{2}\right\}\right)-2f\left(\left\{\frac{ie_4}{2},\frac{ie_6}{2}\right\}\right)+2i f\left(\left\{-\frac{ie_5}{2},-\frac{ie_6}{2}\right\}\right)$
e_3-ie_1
$\mathrm{u}_1^{\mathfrak{g}}\circ\nu=0$

$\mathrm{u}_1^{\mathfrak{b}}$
$-f\left(\left\{-\frac{e_1}{2},-\frac{e_5}{2}\right\}\right)-f\left(\left\{-\frac{ie_1}{2},-\frac{ie_4}{2}\right\}\right)+f\left(\left\{-\frac{ie_3}{2},-\frac{ie_5}{2}\right\}\right)+f\left(\left\{\frac{e_3}{2},\frac{e_4}{2}\right\}\right)$
$-\frac{e_2}{2}$
$\mathrm{u}_1^{\mathfrak{b}}\circ\nu=-\frac{e_2}{4}+\frac{ie_6}{4}$

$\mathbf{e_2}$
$f(\{e_1,e_2,e_3\})+i f(\{e_1,e_3,e_6\})+i f(\{e_2\})+f(\{e_6\})$
$2e_6+2ie_2$
$e_2\circ\nu=2e_6+2ie_2$

$\boldsymbol{\nu_2}$
$f(\{e_1,e_3\})+f(\{e_2,e_6\})+f(\{e_4,e_5\})$
$3e_7$
$\nu_2\circ\nu=\frac{3e_7}{2}-\frac{3i}{2}$

$\mathrm{d}_2^{\mathfrak{r}}$
$f(\{e_1,e_2,e_3\})+i f(\{e_1,e_3,e_6\})+i f(\{e_2\})+f(\{e_6\})$
$2e_6+2ie_2$
$\mathrm{d}_2^{\mathfrak{r}}\circ\nu=2e_6+2ie_2$

$\mathrm{d}_2^{\mathfrak{g}}$
$-i f(\{e_1,e_2,e_4\})-f(\{e_1,e_2,e_5\})+f(\{e_1,e_4,e_6\})-i f(\{e_1,e_5,e_6\})$
$2e_7+2i$
$\mathrm{d}_2^{\mathfrak{g}}\circ\nu=0$

$\mathrm{d}_2^{\mathfrak{b}}$
$f(\{e_1,e_3,e_4\})-i f(\{e_1,e_3,e_5\})-i f(\{e_4\})-f(\{e_5\})$
$-2e_5-2ie_4$
$\mathrm{d}_2^{\mathfrak{b}}\circ\nu=-2e_5-2ie_4$

$\mathrm{u}_2^{\mathfrak{r}}$
$2f\left(\left\{\frac{ie_1}{2},\frac{ie_2}{2}\right\}\right)+2i f\left(\left\{\frac{ie_1}{2},\frac{ie_6}{2}\right\}\right)-2i f\left(\left\{\frac{ie_2}{2},\frac{ie_3}{2}\right\}\right)-2f\left(\left\{\frac{ie_3}{2},\frac{ie_6}{2}\right\}\right)$
$-e_4+ie_5$
$\mathrm{u}_2^{\mathfrak{r}}\circ\nu=-e_4+ie_5$

$\mathrm{u}_2^{\mathfrak{g}}$
$2i f\left(\left\{\frac{ie_2}{2},\frac{ie_4}{2}\right\}\right)+2f\left(\left\{\frac{ie_2}{2},\frac{ie_5}{2}\right\}\right)-2f\left(\left\{\frac{ie_4}{2},\frac{ie_6}{2}\right\}\right)+2i f\left(\left\{-\frac{ie_5}{2},-\frac{ie_6}{2}\right\}\right)$
e_3-ie_1
$\mathrm{u}_2^{\mathfrak{g}}\circ\nu=0$

$\mathrm{u}_2^{\mathfrak{b}}$
$-f\left(\left\{-\frac{e_1}{2},-\frac{e_5}{2}\right\}\right)-f\left(\left\{-\frac{ie_1}{2},-\frac{ie_4}{2}\right\}\right)+f\left(\left\{-\frac{ie_3}{2},-\frac{ie_5}{2}\right\}\right)+f\left(\left\{\frac{e_3}{2},\frac{e_4}{2}\right\}\right)$
$-\frac{e_2}{2}$
$\mathrm{u}_2^{\mathfrak{b}}\circ\nu=-\frac{e_2}{4}+\frac{ie_6}{4}$

$\mathbf{e_3}$
$-f(\{e_1,e_3,e_4\})+i f(\{e_1,e_3,e_5\})+i f(\{e_4\})+f(\{e_5\})$
$2e_5+2ie_4$
$e_3\circ\nu=2e_5+2ie_4$

$\boldsymbol{\nu_3}$
$-i f(\{e_1,e_2,e_4\})-f(\{e_1,e_2,e_5\})-f(\{e_1,e_4,e_6\})-i f(\{e_1,e_5,e_6\})$
$2i$
$\nu_3\circ\nu=-e_7+i$

$\mathrm{d}_3^{\mathfrak{r}}$
$-i f(\{e_1,e_2,e_4\})+f(\{e_1,e_2,e_5\})+f(\{e_1,e_4,e_6\})+i f(\{e_1,e_5,e_6\})$
0
$\mathrm{d}_3^{\mathfrak{r}}\circ\nu=0$

$\mathrm{d}_3^{\mathfrak{g}}$
$f(\{e_1,e_2,e_3\})+i f(\{e_1,e_3,e_6\})-i f(\{e_2\})-f(\{e_6\})$
0
$\mathrm{d}_3^{\mathfrak{g}}\circ\nu=0$

$\mathrm{d}_3^{\mathfrak{b}}$
$f(\{e_1,e_2,e_6\})-f(\{e_1,e_4,e_5\})+i f(\{e_1\})+f(\{e_3\})$
e_3+ie_1
$\mathrm{d}_3^{\mathfrak{b}}\circ\nu=e_3+ie_1$

$\mathrm{u}_3^{\mathfrak{r}}$
$f(\{e_1,e_3,e_4\})+i f(\{e_1,e_3,e_5\})-i f(\{e_4\})+f(\{e_5\})$
0
$\mathrm{u}_3^{\mathfrak{r}}\circ\nu=0$

$\mathrm{u}_3^{\mathfrak{g}}$
$f(\{e_1,e_2,e_6\})+f(\{e_1,e_4,e_5\})-i f(\{e_1\})+f(\{e_3\})$
$-e_3-ie_1$
$\mathrm{u}_3^{\mathfrak{g}}\circ\nu=-e_3-ie_1$

$\mathrm{u}_3^{\mathfrak{b}}$
$-f(\{e_1,e_2,e_3\})+i f(\{e_1,e_3,e_6\})-i f(\{e_2\})+f(\{e_6\})$
0
$\mathrm{u}_3^{\mathfrak{b}}\circ\nu=0$

Visualize the entire family of flavors across 3 generations (2019 version)

These show the reduced octonion structure with pre/post sS, the Q based on the sS used (not sure if this is what is intended), as well as the the unevaluated (string form) showing what I assume is the commutation structure of eqs. 19, 22 and 23 action to derive proper charge Q (which doesn't seem to work at all). I need clarification on how this is supposed to operate (maybe more examples):

In[2739]:=
```
doSM := Column[Table[Column[Table[Row[Table[
        prt = Expand[
          ToExpression[flavorList[[flv]] <> ToString@gen <> If[flv > 2, rgb[[col]], " "]] /. f → fff];
        sprt = ToString[Subsuperscript[flavorList[[flv]], gen, If[flv > 2, rgb[[col]], " "]],
          TraditionalForm];
        ssprt = Style[sprt, Bold, Red];
        sQ = su3Q@prt;
        sQ1 = "Q=" <> ToString[sQ[[2]], TraditionalForm];
        sQ2 = ToString[Row@sQ, TraditionalForm] <> "⊙" <> ToString[prt[[1, 2]], TraditionalForm] <>
          sprt <> ToString[prt[[1, 1]], TraditionalForm] <> "=";
        sQ3 = "i(" <> ToString[sQ[[2]], TraditionalForm] <> ToString[Row@prt[[1]], TraditionalForm] <>
          "⊙" <> ToString[Row@prt[[1]], TraditionalForm] <> sprt <>
          ToString[Row@prt[[3]], TraditionalForm] <> ")=";
        sQ4 = ToString[Row@sQ[[ ;; 2]], TraditionalForm] <> sprt;
        (* skip color iterations on leptons *)
        If[flv < 3 && col > 1, Nothing,
         Column[{ssprt, Row@prt, sQ1, Row@{sQ2, sQ3, sQ4, "      "}}, Center]],
        {col, 3}]],
      {flv, 4}], Center],
    {gen, 3}], Center];
```

In[2740]:= **doSM**

Out[2740]=

$\mathbf{e_1}$
$\{s, S^*\}\{-3\,e_3 - i\,e_1\}\{s^*, S^*\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{e}_1 s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{e}_1 s^*S^*)=i\frac{2}{3}\mathrm{e}_1$

$\boldsymbol{\nu_1}$
$\{s^*, S\}\{1. + 0.\,i\}\{s^*, S\}$
$Q=\frac{1}{3}$
$i\frac{1}{3}s^*\odot S\nu_1 s^*=i(\frac{1}{3}s^*S\odot s^*S\nu_1 s^*S)=i\frac{1}{3}\nu_1$

$\mathbf{d_1^r}$
$\{s, S^*\}\{-e_3 - i\,e_1\}\{s^*, S\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{d}_1^r s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{d}_1^r s^*S)=i\frac{2}{3}\mathrm{d}_1^r$

$\mathbf{d_1^g}$
$\{s, S^*\}\{0\}\{s^*, S\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{d}_1^g s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{d}_1^g s^*S)=i\frac{2}{3}\mathrm{d}_1^g$

$\mathbf{d_1^b}$
$\{s, S^*\}\{0\}\{s^*, S\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{d}_1^b s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{d}_1^b s^*S)=i\frac{2}{3}\mathrm{d}_1^b$

$\mathbf{u_1^r}$
$\{s, S^*\}\{-e_4 + i\,e_5\}\{s, S\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{u}_1^r s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{u}_1^r sS)=i\frac{2}{3}\mathrm{u}_1^r$

$\mathbf{u_1^g}$
$\{s, S^*\}\{e_3 - i\,e_1\}\{s, S\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{u}_1^g s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{u}_1^g sS)=i\frac{2}{3}\mathrm{u}_1^g$

$\mathbf{u_1^b}$
$\{s, S^*\}\{-\frac{e_2}{2}\}\{s, S\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{u}_1^b s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{u}_1^b sS)=i\frac{2}{3}\mathrm{u}_1^b$

$\mathbf{e_2}$
$\{s, S^*\}\{2\,e_6 + 2\,i\,e_2\}\{s^*, S^*\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{e}_2 s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{e}_2 s^*S^*)=i\frac{2}{3}\mathrm{e}_2$

$\boldsymbol{\nu_2}$
$\{s, S^*\}\{3\,e_7\}\{s, S^*\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\nu_2 s=i(\frac{2}{3}sS^*\odot sS^*\nu_2 sS^*)=i\frac{2}{3}\nu_2$

$\mathbf{d_2^r}$
$\{s, S^*\}\{2\,e_6 + 2\,i\,e_2\}\{s^*, S\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{d}_2^r s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{d}_2^r s^*S)=i\frac{2}{3}\mathrm{d}_2^r$

$\mathbf{d_2^g}$
$\{s, S^*\}\{2\,e_7 + 2\,i\}\{s^*, S\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{d}_2^g s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{d}_2^g s^*S)=i\frac{2}{3}\mathrm{d}_2^g$

$\mathbf{d_2^b}$
$\{s, S^*\}\{-2\,e_5 - 2\,i\,e_4\}\{s^*, S\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{d}_2^b s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{d}_2^b s^*S)=i\frac{2}{3}\mathrm{d}_2^b$

$\mathbf{u_2^r}$
$\{s^*, S^*\}\{-e_4 + i\,e_5\}\{s^*, S\}$
$Q=1$
$i1s^*\odot S^*\mathrm{u}_2^r s^*=i(1s^*S^*\odot s^*S^*\mathrm{u}_2^r s^*S)=i1\mathrm{u}_2^r$

$\mathbf{u_2^g}$
$\{s^*, S^*\}\{e_3 - i\,e_1\}\{s^*, S\}$
$Q=1$
$i1s^*\odot S^*\mathrm{u}_2^g s^*=i(1s^*S^*\odot s^*S^*\mathrm{u}_2^g s^*S)=i1\mathrm{u}_2^g$

$\mathbf{u_2^b}$
$\{s^*, S^*\}\{-\frac{e_2}{2}\}\{s^*, S\}$
$Q=1$
$i1s^*\odot S^*\mathrm{u}_2^b s^*=i(1s^*S^*\odot s^*S^*\mathrm{u}_2^b s^*S)=i1\mathrm{u}_2^b$

$\mathbf{e_3}$
$\{s, S^*\}\{2\,e_5 + 2\,i\,e_4\}\{s^*, S^*\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{e}_3 s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{e}_3 s^*S^*)=i\frac{2}{3}\mathrm{e}_3$

$\boldsymbol{\nu_3}$
$\{s^*, S^*\}\{2\,i\}\{s, S\}$
$Q=1$
$i1s^*\odot S^*\nu_3 s^*=i(1s^*S^*\odot s^*S^*\nu_3 sS)=i1\nu_3$

$\mathbf{d_3^r}$
$\{s, S^*\}\{0\}\{s^*, S\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{d}_3^r s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{d}_3^r s^*S)=i\frac{2}{3}\mathrm{d}_3^r$

$\mathbf{d_3^g}$
$\{s, S^*\}\{0\}\{s^*, S\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{d}_3^g s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{d}_3^g s^*S)=i\frac{2}{3}\mathrm{d}_3^g$

$\mathbf{d_3^b}$
$\{s, S^*\}\{e_3 + i\,e_1\}\{s^*, S\}$
$Q=\frac{2}{3}$
$i\frac{2}{3}s\odot S^*\mathrm{d}_3^b s=i(\frac{2}{3}sS^*\odot sS^*\mathrm{d}_3^b s^*S)=i\frac{2}{3}\mathrm{d}_3^b$

$\mathbf{u_3^r}$
$\{s, S\}\{0\}\{s^*, S\}$
$Q=\frac{4}{3}$
$i\frac{4}{3}s\odot S\mathrm{u}_3^r s=i(\frac{4}{3}sS\odot sS\mathrm{u}_3^r s^*S)=i\frac{4}{3}\mathrm{u}_3^r$

$\mathbf{u_3^g}$
$\{s, S\}\{-e_3 - i\,e_1\}\{s^*, S\}$
$Q=\frac{4}{3}$
$i\frac{4}{3}s\odot S\mathrm{u}_3^g s=i(\frac{4}{3}sS\odot sS\mathrm{u}_3^g s^*S)=i\frac{4}{3}\mathrm{u}_3^g$

$\mathbf{u_3^b}$
$\{s, S\}\{0\}\{s^*, S\}$
$Q=\frac{4}{3}$
$i\frac{4}{3}s\odot S\mathrm{u}_3^b s=i(\frac{4}{3}sS\odot sS\mathrm{u}_3^b s^*S)=i\frac{4}{3}\mathrm{u}_3^b$

Checking all SM particle color and flavor changing actions (e.g. $\Lambda_n[v] \odot$u1g) using 4 forms of $\Lambda_{n=1-8}$ and $g_{n=9-14}$ (*i.e.* with $f \to v$, $i\!\!\!i\, v$, v^*, $v^\dagger$)

The output is extensive and given my open questions on the formalism presented, the accuracy likely deviates from the intent of [5], but it is interesting to show how everything transforms. If no transform is found for a particular action, it outputs an * for that action. If a color or flavor changing transformation action is found, it identifies that action with the list of particles the transformation applies to. Note: it only identifies a transformed particle if the source particle has a non-zero reduced value and the resulting match is exact (red) or a ±integer factor of that particle (blue).

In[2741]:=
```
Λglist = {"v", "ⅈ v", "v*", "v†"};
```

In[2742]:=
```
(* This generates a list of particle reductions
 used to compare color and flavor changes against *)
listSM = Flatten[Table[Table[Table[Table[
       prt = ToExpression[flavorList[[flv]] <> ToString@gen <> If[flv > 2, rgb[[col]], " "]];
       (* Conjugate if anti=True *)
       If[anti, prt = prt*];
       sprt = ToString[
         Subsuperscript[flavorList[[flv]], gen, If[flv > 2, rgb[[col]], " "]], TraditionalForm];
       If[anti, sprt = OverBar[sprt]];
       ssprt = Style[sprt, Bold, Red];
       (* skip color iterations on leptons *)
       If[flv < 3 && col > 1, Nothing,
        {sprt, Expand[prt[[2, 1]] /. f → fff]}],
       {col, 3}],
      {flv, 4}],
     {gen, 3}],
    {anti, {False, True}}], 3]
```

Out[2742]=

$$\begin{pmatrix}
\mathrm{e}_1 & -i\,e_1 - 3\,e_3 \\
\nu_1 & 1. + 0.\,i \\
\mathrm{d}_1^{\mathfrak{r}} & -i\,e_1 - e_3 \\
\mathrm{d}_1^{\mathfrak{g}} & 0 \\
\mathrm{d}_1^{\mathfrak{b}} & 0 \\
\mathrm{u}_1^{\mathfrak{r}} & i\,e_5 - e_4 \\
\mathrm{u}_1^{\mathfrak{g}} & e_3 - i\,e_1 \\
\mathrm{u}_1^{\mathfrak{b}} & -\frac{e_2}{2} \\
\mathrm{e}_2 & 2\,i\,e_2 + 2\,e_6 \\
\nu_2 & 3\,e_7 \\
\mathrm{d}_2^{\mathfrak{r}} & 2\,i\,e_2 + 2\,e_6 \\
\mathrm{d}_2^{\mathfrak{g}} & 2\,e_7 + 2\,i \\
\mathrm{d}_2^{\mathfrak{b}} & -2\,i\,e_4 - 2\,e_5 \\
\mathrm{u}_2^{\mathfrak{r}} & i\,e_5 - e_4 \\
\mathrm{u}_2^{\mathfrak{g}} & e_3 - i\,e_1 \\
\mathrm{u}_2^{\mathfrak{b}} & -\frac{e_2}{2} \\
\mathrm{e}_3 & 2\,i\,e_4 + 2\,e_5 \\
\nu_3 & 2\,i \\
\mathrm{d}_3^{\mathfrak{r}} & 0 \\
\mathrm{d}_3^{\mathfrak{g}} & 0 \\
\mathrm{d}_3^{\mathfrak{b}} & i\,e_1 + e_3 \\
\mathrm{u}_3^{\mathfrak{r}} & 0 \\
\mathrm{u}_3^{\mathfrak{g}} & -i\,e_1 - e_3 \\
\mathrm{u}_3^{\mathfrak{b}} & 0 \\
\overline{\mathrm{e}_1} & i\,e_1 - 3\,e_3 \\
\overline{\nu_1} & 1. + 0.\,i \\
\overline{\mathrm{d}_1^{\mathfrak{r}}} & i\,e_1 - e_3 \\
\overline{\mathrm{d}_1^{\mathfrak{g}}} & 0 \\
\overline{\mathrm{d}_1^{\mathfrak{b}}} & 0 \\
\overline{\mathrm{u}_1^{\mathfrak{r}}} & -e_4 - i\,e_5 \\
\overline{\mathrm{u}_1^{\mathfrak{g}}} & i\,e_1 + e_3 \\
\overline{\mathrm{u}_1^{\mathfrak{b}}} & -\frac{e_2}{2} \\
\overline{\mathrm{e}_2} & 2\,e_6 - 2\,i\,e_2 \\
\overline{\nu_2} & 3\,e_7 \\
\overline{\mathrm{d}_2^{\mathfrak{r}}} & 2\,e_6 - 2\,i\,e_2 \\
\overline{\mathrm{d}_2^{\mathfrak{g}}} & 2\,e_7 - 2\,i \\
\overline{\mathrm{d}_2^{\mathfrak{b}}} & 2\,i\,e_4 - 2\,e_5 \\
\overline{\mathrm{u}_2^{\mathfrak{r}}} & -e_4 - i\,e_5 \\
\overline{\mathrm{u}_2^{\mathfrak{g}}} & i\,e_1 + e_3 \\
\overline{\mathrm{u}_2^{\mathfrak{b}}} & -\frac{e_2}{2} \\
\overline{\mathrm{e}_3} & 2\,e_5 - 2\,i\,e_4 \\
\overline{\nu_3} & -2\,i \\
\overline{\mathrm{d}_3^{\mathfrak{r}}} & 0 \\
\overline{\mathrm{d}_3^{\mathfrak{g}}} & 0 \\
\overline{\mathrm{d}_3^{\mathfrak{b}}} & e_3 - i\,e_1 \\
\overline{\mathrm{u}_3^{\mathfrak{r}}} & 0 \\
\overline{\mathrm{u}_3^{\mathfrak{g}}} & i\,e_1 - e_3 \\
\overline{\mathrm{u}_3^{\mathfrak{b}}} & 0
\end{pmatrix}$$

In[2743]:=
```
(* This checks one non-zero reduced commutation result (e.g. Λn[ν]⊙u1g) against all of the non-
 zero reduced particle values generated in lisSM *)
chkRes@res_ := Module[{res2, pos1p, pos2p, pos3p, pos1m, pos2m, pos3m, pos3pos, pos, pos1, fl},
   (* Generate the postion lists for each comparison *)
   res2 = Numerator@# & /@ Simplify[(**)res];
   pos1p = Position[listSM⟦All, 2⟧, Expand[1 res2]];
   pos2p = Position[listSM⟦All, 2⟧, Expand[2 res2]];
   pos3p = Position[listSM⟦All, 2⟧, Expand[3 res2]];
   pos1m = Position[listSM⟦All, 2⟧, Expand[-1 res2]];
   pos2m = Position[listSM⟦All, 2⟧, Expand[-2 res2]];
   pos3m = Position[listSM⟦All, 2⟧, Expand[-3 res2]];
   fl = {pos1p, pos2p, pos3p, pos1m, pos2m, pos3m};
   pos = DeleteDuplicates@Flatten@fl;
   pos1 = DeleteDuplicates@Flatten@pos1p;
   (* If the input is not zero and there was a position found,
   output that particle - color code for red as exact match and blue is a ±integer factor *)
   If[res =!= 0 && Length@pos > 0,
    Style[{listSM⟦Flatten@pos, 1⟧, res},
     If[Length@pos1p > 0 && MemberQ[listSM⟦pos1, 2⟧, res], Red, Blue]],
    Style[{"*", res}, Black]]];
```

In[2744]:=
```
(* This generates all of the possible combinations of color
 and flavor commutations across the SM and lists the checked results *)
doSMactionAll = Module[{prt, sprt, ssprt, pos, res, in},
  Column[Table[Column[Table[Column[Table[Row@Table[
        prt = ToExpression[flavorList⟦flv⟧ <> ToString@gen <> If[flv > 2, rgb⟦col⟧, " "]];
        (* Conjugate if anti=True *)
        If[anti, prt = prt*];
        sprt = ToString[
          Subsuperscript[flavorList⟦flv⟧, gen, If[flv > 2, rgb⟦col⟧, " "]], TraditionalForm];
        If[anti, sprt = ToString[sprt (overbar), TraditionalForm]];
        ssprt = Style[sprt, Bold, Red];
        (* skip color iterations on leptons *)
        If[flv < 3 && col > 1, Nothing,
         Column[{ssprt,
           Row@{sprt <> "=    ", Row@{Expand@prt⟦2, 1⟧ /. f → fff}},
           MatrixForm@Join[
             Table[in = "Λ[" <> ToString[#] <> "," <> Λglist⟦Λg⟧ <> "]";
                res = doAction[ToExpression@in, prt⟦2, 1⟧];
                Row@{in, chkRes@res},
                {Λg, Length@Λglist}] & /@ Range@8,
             Table[in = "g[" <> ToString[#] <> "," <> Λglist⟦Λg⟧ <> "]";
                res = doAction[ToExpression@in, prt⟦2, 1⟧];
                Row@{in, chkRes@res},
                {Λg, Length@Λglist}] & /@ Range[9, 14]]}, Center]],
        {col, 3}],
       {flv, 4}], Center],
     {gen, 3}], Center],
   {anti, {False, True}}],
  Center]]
```

$\mathbf{e_1}$

$e_1 = -3e_3 - i e_1$

$\Lambda[1,\nu]\{\{u_1^r, u_2^r\}, e_4 - i e_5\}$	$\Lambda[1,i\nu]\{\{e_3, d_2^b\}, e_5 + i e_4\}$	$\Lambda[1,\nu^*]\{*, 2e_4 + 2ie_5\}$	$\Lambda[1,\nu^\dagger]\{\{u_1^r, u_2^r\}, -e_4 + i e_5\}$
$\Lambda[2,\nu]\{*, 0\}$	$\Lambda[2,i\nu]\{*, 0\}$	$\Lambda[2,\nu^*]\{*, 0\}$	$\Lambda[2,\nu^\dagger]\{*, 0\}$
$\Lambda[3,\nu]\{*, -\frac{3e_1}{2} + \frac{ie_3}{2}\}$	$\Lambda[3,i\nu]\{*, -\frac{e_3}{2} - \frac{3ie_1}{2}\}$	$\Lambda[3,\nu^*]\{*, -\frac{3e_1}{2} + \frac{ie_3}{2}\}$	$\Lambda[3,\nu^\dagger]\{*, \frac{3e_1}{2} - \frac{ie_3}{2}\}$
$\Lambda[4,\nu]\{*, 0\}$	$\Lambda[4,i\nu]\{*, 0\}$	$\Lambda[4,\nu^*]\{*, 0\}$	$\Lambda[4,\nu^\dagger]\{*, 0\}$
$\Lambda[5,\nu]\{\{d_2^g, \nu_1, \overline{d_2^g}\}, 2e_7\}$	$\Lambda[5,i\nu]\{*, 2ie_7\}$	$\Lambda[5,\nu^*]\{\{\nu_2, \nu_1, d_2^g, \overline{\nu_2}, \overline{d_2^g}\}, e_7\}$	$\Lambda[5,\nu^\dagger]\{\{d_2^g, \nu_1, \overline{d_2^g}\}, -2e_7\}$
$\Lambda[6,\nu]\{*, 0\}$	$\Lambda[6,i\nu]\{*, 0\}$	$\Lambda[6,\nu^*]\{*, 0\}$	$\Lambda[6,\nu^\dagger]\{*, 0\}$
$\Lambda[7,\nu]\{*, 0\}$	$\Lambda[7,i\nu]\{*, 0\}$	$\Lambda[7,\nu^*]\{*, 0\}$	$\Lambda[7,\nu^\dagger]\{*, 0\}$
$\Lambda[8,\nu]\{*, -\frac{3e_1}{2} + \frac{ie_3}{2}\}$	$\Lambda[8,i\nu]\{*, -\frac{e_3}{2} - \frac{3ie_1}{2}\}$	$\Lambda[8,\nu^*]\{*, -\frac{3e_1}{2} + \frac{ie_3}{2}\}$	$\Lambda[8,\nu^\dagger]\{*, \frac{3e_1}{2} - \frac{ie_3}{2}\}$
$g[9,\nu]\{\{\overline{u_1^r}, \overline{u_2^r}\}, \frac{e_4}{36} + \frac{ie_5}{36}\}$	$g[9,i\nu]\{\{\overline{d_2^b}, \overline{e_3}\}, -\frac{e_5}{36} + \frac{ie_4}{36}\}$	$g[9,\nu^*]\{\{u_1^r, u_2^r\}, \frac{e_4}{72} - \frac{ie_5}{72}\}$	$g[9,\nu^\dagger]\{\{\overline{u_1^r}, \overline{u_2^r}\}, -\frac{e_4}{36} - \frac{ie_5}{36}\}$
$g[10,\nu]\{\{\overline{d_2^b}, \overline{e_3}\}, -\frac{e_5}{36} + \frac{ie_4}{36}\}$	$g[10,i\nu]\{\{\overline{u_1^r}, \overline{u_2^r}\}, -\frac{e_4}{36} - \frac{ie_5}{36}\}$	$g[10,\nu^*]\{\{d_2^b, e_3\}, -\frac{e_5}{72} - \frac{ie_4}{72}\}$	$g[10,\nu^\dagger]\{\{\overline{e_3}, \overline{d_2^b}\}, \frac{e_5}{36} - \frac{ie_4}{36}\}$
$g[11,\nu]\{*, -\frac{ie_7}{72}\}$	$g[11,i\nu]\{\{\nu_2, \nu_1, d_2^g, \overline{\nu_2}, \overline{d_2^g}\}, \frac{e_7}{72}\}$	$g[11,\nu^*]\{*, \frac{ie_7}{36}\}$	$g[11,\nu^\dagger]\{*, \frac{ie_7}{72}\}$
$g[12,\nu]\{\{\nu_2, \nu_1, d_2^g, \overline{\nu_2}, \overline{d_2^g}\}, -\frac{e_7}{72}\}$	$g[12,i\nu]\{*, -\frac{ie_7}{72}\}$	$g[12,\nu^*]\{\{\nu_2, \nu_1, d_2^g, \overline{\nu_2}, \overline{d_2^g}\}, -\frac{e_7}{36}\}$	$g[12,\nu^\dagger]\{\{\nu_2, \nu_1, d_2^g, \overline{\nu_2}, \overline{d_2^g}\}, \frac{e_7}{72}\}$
$g[13,\nu]\{*, \frac{e_2}{12} - \frac{ie_6}{12}\}$	$g[13,i\nu]\{\{e_2, d_2^r\}, \frac{e_6}{12} + \frac{ie_2}{12}\}$	$g[13,\nu^*]\{*, \frac{e_2}{6} + \frac{ie_6}{6}\}$	$g[13,\nu^\dagger]\{*, -\frac{e_2}{12} + \frac{ie_6}{12}\}$
$g[14,\nu]\{\{\overline{e_2}, \overline{d_2^r}\}, \frac{e_6}{36} - \frac{ie_2}{36}\}$	$g[14,i\nu]\{*, \frac{e_2}{36} + \frac{ie_6}{36}\}$	$g[14,\nu^*]\{\{e_2, d_2^r\}, \frac{e_6}{72} + \frac{ie_2}{72}\}$	$g[14,\nu^\dagger]\{\{\overline{e_2}, \overline{d_2^r}\}, -\frac{e_6}{36} + \frac{ie_2}{36}\}$

$\mathbf{\nu_1}$

$\nu_1 = 1 + 0\, i$

Λ[1,ν]{*, 0}	Λ[1,i ν]{*, 0}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,i ν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,i ν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{*, 0}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{*, 0}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,i ν]{*, 0}	g[11,ν*]{*, 0}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,i ν]{*, 0}	g[12,ν*]{*, 0}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, 0}	g[13,ν†]{*, 0}
g[14,ν]{*, 0}	g[14,i ν]{*, 0}	g[14,ν*]{*, 0}	g[14,ν†]{*, 0}

d_1^r

$d_1^r = -e_3 - i e_1$

Λ[1,ν]{*, 0}	Λ[1,i ν]{*, 0}	Λ[1,ν*]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $e_4 + i e_5$}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, $-\frac{e_1}{2} + \frac{i e_3}{2}$}	Λ[3,i ν]{{d_1^r, u_3^g, d_3^b, $\overline{u_1^g}$, $\overline{u_2^g}$}, $-\frac{e_3}{2} - \frac{i e_1}{2}$}	Λ[3,ν*]{*, $-\frac{e_1}{2} + \frac{i e_3}{2}$}	Λ[3,ν†]{*, $\frac{e_1}{2} - \frac{i e_3}{2}$}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, e_7}	Λ[5,i ν]{*, $i e_7$}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $-e_7$}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, $-\frac{e_1}{2} + \frac{i e_3}{2}$}	Λ[8,i ν]{{d_1^r, u_3^g, d_3^b, $\overline{u_1^g}$, $\overline{u_2^g}$}, $-\frac{e_3}{2} - \frac{i e_1}{2}$}	Λ[8,ν*]{*, $-\frac{e_1}{2} + \frac{i e_3}{2}$}	Λ[8,ν†]{*, $\frac{e_1}{2} - \frac{i e_3}{2}$}
g[9,ν]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $\frac{e_4}{72} + \frac{i e_5}{72}$}	g[9,i ν]{{$\overline{d_2^b}$, $\overline{e_3}$}, $-\frac{e_5}{72} + \frac{i e_4}{72}$}	g[9,ν*]{*, 0}	g[9,ν†]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{72} - \frac{i e_5}{72}$}
g[10,ν]{{$\overline{d_2^b}$, $\overline{e_3}$}, $-\frac{e_5}{72} + \frac{i e_4}{72}$}	g[10,i ν]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{72} - \frac{i e_5}{72}$}	g[10,ν*]{*, 0}	g[10,ν†]{{$\overline{e_3}$, $\overline{d_2^b}$}, $\frac{e_5}{72} - \frac{i e_4}{72}$}
g[11,ν]{*, 0}	g[11,i ν]{*, 0}	g[11,ν*]{*, $\frac{i e_7}{72}$}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,i ν]{*, 0}	g[12,ν*]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $-\frac{e_7}{72}$}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, $\frac{e_2}{12} + \frac{i e_6}{12}$}	g[13,ν†]{*, 0}
g[14,ν]{{$\overline{e_2}$, $\overline{d_2^r}$}, $\frac{e_6}{72} - \frac{i e_2}{72}$}	g[14,i ν]{*, $\frac{e_2}{72} + \frac{i e_6}{72}$}	g[14,ν*]{*, 0}	g[14,ν†]{{$\overline{e_2}$, $\overline{d_2^r}$}, $-\frac{e_6}{72} + \frac{i e_2}{72}$}

d_2^g

$d_2^g = 0$

Λ[1,ν]{*, 0}	Λ[1,i ν]{*, 0}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,i ν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,i ν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{*, 0}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{*, 0}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,i ν]{*, 0}	g[11,ν*]{*, 0}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,i ν]{*, 0}	g[12,ν*]{*, 0}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, 0}	g[13,ν†]{*, 0}
g[14,ν]{*, 0}	g[14,i ν]{*, 0}	g[14,ν*]{*, 0}	g[14,ν†]{*, 0}

d_3^g

$d_3^g = 0$

Λ[1,ν]{*, 0}	Λ[1,i ν]{*, 0}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,i ν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,i ν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{*, 0}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{*, 0}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,i ν]{*, 0}	g[11,ν*]{*, 0}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,i ν]{*, 0}	g[12,ν*]{*, 0}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, 0}	g[13,ν†]{*, 0}
g[14,ν]{*, 0}	g[14,i ν]{*, 0}	g[14,ν*]{*, 0}	g[14,ν†]{*, 0}

u_1^r

$u_1^r = -e_4 + i e_5$

Λ[1,ν]{*, 0}	Λ[1,i ν]{*, 0}	Λ[1,ν*]{{$\overline{d_1^r}$, $\overline{u_3^g}$, u_1^g, u_2^g, $\overline{d_3^b}$}, $-e_3 + i e_1$}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{{e_3, d_2^b}, $\frac{e_5}{2} + \frac{i e_4}{2}$}	Λ[3,i ν]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{2} + \frac{i e_5}{2}$}	Λ[3,ν*]{{e_3, d_2^b}, $\frac{e_5}{2} + \frac{i e_4}{2}$}	Λ[3,ν†]{{$\overline{d_2^b}$, $\overline{e_3}$}, $-\frac{e_5}{2} - \frac{i e_4}{2}$}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{*, $-e_2 - i e_6$}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{{e_3, d_2^b}, $\frac{e_5}{2} + \frac{i e_4}{2}$}	Λ[8,i ν]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{2} + \frac{i e_5}{2}$}	Λ[8,ν*]{{e_3, d_2^b}, $\frac{e_5}{2} + \frac{i e_4}{2}$}	Λ[8,ν†]{{$\overline{d_2^b}$, $\overline{e_3}$}, $-\frac{e_5}{2} - \frac{i e_4}{2}$}
g[9,ν]{{u_1^g, u_2^g, $\overline{d_3^b}$, $\overline{d_1^r}$, $\overline{u_3^g}$}, $-\frac{e_3}{72} + \frac{i e_1}{72}$}	g[9,i ν]{*, $-\frac{e_1}{72} - \frac{i e_3}{72}$}	g[9,ν*]{*, 0}	g[9,ν†]{{u_1^g, u_2^g, $\overline{d_3^b}$, $\overline{d_1^r}$, $\overline{u_3^g}$}, $\frac{e_3}{72} - \frac{i e_1}{72}$}
g[10,ν]{*, $-\frac{e_1}{72} - \frac{i e_3}{72}$}	g[10,i ν]{{u_1^g, u_2^g, $\overline{d_3^b}$, $\overline{d_1^r}$, $\overline{u_3^g}$}, $\frac{e_3}{72} - \frac{i e_1}{72}$}	g[10,ν*]{*, 0}	g[10,ν†]{*, $\frac{e_1}{72} + \frac{i e_3}{72}$}
g[11,ν]{{$\overline{e_2}$, $\overline{d_2^r}$}, $-\frac{e_6}{72} + \frac{i e_2}{72}$}	g[11,i ν]{*, $-\frac{e_2}{72} - \frac{i e_6}{72}$}	g[11,ν*]{*, 0}	g[11,ν†]{{$\overline{e_2}$, $\overline{d_2^r}$}, $\frac{e_6}{72} - \frac{i e_2}{72}$}
g[12,ν]{*, $\frac{e_2}{72} + \frac{i e_6}{72}$}	g[12,i ν]{{$\overline{e_2}$, $\overline{d_2^r}$}, $-\frac{e_6}{72} + \frac{i e_2}{72}$}	g[12,ν*]{*, 0}	g[12,ν†]{*, $-\frac{e_2}{72} - \frac{i e_6}{72}$}
g[13,ν]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $\frac{e_7}{12}$}	g[13,i ν]{*, $\frac{i e_7}{12}$}	g[13,ν*]{*, 0}	g[13,ν†]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $-\frac{e_7}{12}$}
g[14,ν]{*, 0}	g[14,i ν]{*, 0}	g[14,ν*]{*, $\frac{i e_7}{72}$}	g[14,ν†]{*, 0}

u_2^g

$u_2^g = e_3 - i e_1$

Λ[1,ν]{{u_1^r, u_2^r}, $-e_4 + i e_5$}	Λ[1,i ν]{{$\overline{d_2^b}$, e_3}, $-e_5 - i e_4$}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{{u_1^r, u_2^r}, $e_4 - i e_5$}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, $\frac{e_1}{2} + \frac{i e_3}{2}$}	Λ[3,i ν]{{$\overline{d_1^r}$, u_3^g, u_1^g, u_2^g, $\overline{d_3^b}$}, $-\frac{e_3}{2} + \frac{i e_1}{2}$}	Λ[3,ν*]{*, $\frac{e_1}{2} + \frac{i e_3}{2}$}	Λ[3,ν†]{*, $-\frac{e_1}{2} - \frac{i e_3}{2}$}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $-e_7$}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, $\frac{e_1}{2} + \frac{i e_3}{2}$}	Λ[8,i ν]{{$\overline{d_1^r}$, u_3^g, u_1^g, u_2^g, $\overline{d_3^b}$}, $-\frac{e_3}{2} + \frac{i e_1}{2}$}	Λ[8,ν*]{*, $\frac{e_1}{2} + \frac{i e_3}{2}$}	Λ[8,ν†]{*, $-\frac{e_1}{2} - \frac{i e_3}{2}$}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{72} + \frac{i e_5}{72}$}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{{e_3, d_2^b}, $\frac{e_5}{72} + \frac{i e_4}{72}$}	g[10,ν†]{*, 0}
g[11,ν]{*, $\frac{i e_7}{72}$}	g[11,i ν]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $-\frac{e_7}{72}$}	g[11,ν*]{*, 0}	g[11,ν†]{*, $-\frac{i e_7}{72}$}
g[12,ν]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $\frac{e_7}{72}$}	g[12,i ν]{*, $\frac{i e_7}{72}$}	g[12,ν*]{*, 0}	g[12,ν†]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $-\frac{e_7}{72}$}
g[13,ν]{*, $-\frac{e_2}{12} + \frac{i e_6}{12}$}	g[13,i ν]{{$\overline{e_2}$, $\overline{d_2^r}$}, $-\frac{e_6}{12} - \frac{i e_2}{12}$}	g[13,ν*]{*, 0}	g[13,ν†]{*, $\frac{e_2}{12} - \frac{i e_6}{12}$}
g[14,ν]{*, 0}	g[14,i ν]{*, 0}	g[14,ν*]{{$\overline{e_2}$, $\overline{d_2^r}$}, $-\frac{e_6}{72} - \frac{i e_2}{72}$}	g[14,ν†]{*, 0}

$\nu_1^\dagger$

$\nu_1^\dagger = -\frac{e_2}{2}$

Λ[1,ν]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $-\frac{e_7}{4}$}	Λ[1,i ν]{*, $-\frac{i e_7}{4}$}	Λ[1,ν*]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $-\frac{e_7}{4}$}	Λ[1,ν†]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $\frac{e_7}{4}$}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{{e_2, ν_1, d_2^r, $\overline{e_2}$, $\overline{d_2^r}$}, $\frac{e_6}{4}$}	Λ[3,i ν]{*, $\frac{i e_6}{4}$}	Λ[3,ν*]{{e_2, ν_1, d_2^r, $\overline{e_2}$, $\overline{d_2^r}$}, $\frac{e_6}{4}$}	Λ[3,ν†]{{e_2, ν_1, d_2^r, $\overline{e_2}$, $\overline{d_2^r}$}, $-\frac{e_6}{4}$}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $\frac{e_4}{4} - \frac{i e_5}{4}$}	Λ[5,i ν]{{$\overline{e_3}$, $\overline{d_2^b}$}, $\frac{e_5}{4} + \frac{i e_4}{4}$}	Λ[5,ν*]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $\frac{e_4}{4} + \frac{i e_5}{4}$}	Λ[5,ν†]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{4} + \frac{i e_5}{4}$}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{{e_2, ν_1, d_2^r, $\overline{e_2}$, $\overline{d_2^r}$}, $\frac{e_6}{4}$}	Λ[8,i ν]{*, $\frac{i e_6}{4}$}	Λ[8,ν*]{{e_2, ν_1, d_2^r, $\overline{e_2}$, $\overline{d_2^r}$}, $\frac{e_6}{4}$}	Λ[8,ν†]{{e_2, ν_1, d_2^r, $\overline{e_2}$, $\overline{d_2^r}$}, $-\frac{e_6}{4}$}
g[9,ν]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $-\frac{e_7}{288}$}	g[9,i ν]{*, $-\frac{i e_7}{288}$}	g[9,ν*]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $-\frac{e_7}{288}$}	g[9,ν†]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $\frac{e_7}{288}$}
g[10,ν]{*, $-\frac{i e_7}{288}$}	g[10,i ν]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $\frac{e_7}{288}$}	g[10,ν*]{*, $\frac{i e_7}{288}$}	g[10,ν†]{*, $\frac{i e_7}{288}$}
g[11,ν]{{$\overline{e_3}$, $\overline{d_2^b}$}, $\frac{e_5}{288} - \frac{i e_4}{288}$}	g[11,i ν]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $\frac{e_4}{288} + \frac{i e_5}{288}$}	g[11,ν*]{{e_3, d_2^b}, $\frac{e_5}{288} + \frac{i e_4}{288}$}	g[11,ν†]{{$\overline{d_2^b}$, $\overline{e_3}$}, $-\frac{e_5}{288} + \frac{i e_4}{288}$}
g[12,ν]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{288} - \frac{i e_5}{288}$}	g[12,i ν]{{$\overline{e_3}$, $\overline{d_2^b}$}, $\frac{e_5}{288} - \frac{i e_4}{288}$}	g[12,ν*]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{288} + \frac{i e_5}{288}$}	g[12,ν†]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $\frac{e_4}{288} + \frac{i e_5}{288}$}
g[13,ν]{{u_1^g, u_2^g, $\overline{d_3^b}$, $\overline{d_1^r}$, $\overline{u_3^g}$}, $-\frac{e_3}{48} + \frac{i e_1}{48}$}	g[13,i ν]{*, $\frac{e_1}{48} - \frac{i e_3}{48}$}	g[13,ν*]{{u_1^g, u_2^g, $\overline{d_3^b}$, $\overline{d_1^r}$, $\overline{u_3^g}$}, $-\frac{e_3}{48} + \frac{i e_1}{48}$}	g[13,ν†]{{d_3^b, $\overline{u_1^g}$, $\overline{u_2^g}$, d_1^r, u_3^g}, $\frac{e_3}{48} + \frac{i e_1}{48}$}
g[14,ν]{*, $\frac{e_1}{288} + \frac{i e_3}{288}$}	g[14,i ν]{{u_1^g, u_2^g, $\overline{d_3^b}$, $\overline{d_1^r}$, $\overline{u_3^g}$}, $-\frac{e_3}{288} + \frac{i e_1}{288}$}	g[14,ν*]{*, $\frac{e_1}{288} - \frac{i e_3}{288}$}	g[14,ν†]{*, $-\frac{e_1}{288} - \frac{i e_3}{288}$}

e_2

$e_2 = 2 e_6 + 2 i e_2$

Λ[1,ν]{*, $2 i e_7$}	Λ[1,i ν]{{d_2^g, ν_1, $\overline{d_2^g}$}, $-2 e_7$}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, $-2 i e_7$}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, $e_2 - i e_6$}	Λ[3,i ν]{{e_2, d_2^r}, $e_6 + i e_2$}	Λ[3,ν*]{*, $e_2 - i e_6$}	Λ[3,ν†]{*, $-e_2 + i e_6$}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{{$\overline{e_3}$, $\overline{d_2^b}$}, $2 e_5 - 2 i e_4$}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, $e_2 - i e_6$}	Λ[8,i ν]{{e_2, d_2^r}, $e_6 + i e_2$}	Λ[8,ν*]{*, $e_2 - i e_6$}	Λ[8,ν†]{*, $-e_2 + i e_6$}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{*, $\frac{i e_7}{36}$}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $\frac{e_7}{36}$}	g[10,ν†]{*, 0}
g[11,ν]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{36} - \frac{i e_5}{36}$}	g[11,i ν]{{$\overline{e_3}$, $\overline{d_2^b}$}, $\frac{e_5}{36} - \frac{i e_4}{36}$}	g[11,ν*]{*, 0}	g[11,ν†]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $\frac{e_4}{36} + \frac{i e_5}{36}$}
g[12,ν]{{$\overline{d_2^b}$, $\overline{e_3}$}, $-\frac{e_5}{36} + \frac{i e_4}{36}$}	g[12,i ν]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{36} - \frac{i e_5}{36}$}	g[12,ν*]{*, 0}	g[12,ν†]{{$\overline{e_3}$, $\overline{d_2^b}$}, $\frac{e_5}{36} - \frac{i e_4}{36}$}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, $\frac{e_1}{6} + \frac{i e_3}{6}$}	g[13,ν†]{*, 0}
g[14,ν]{{u_1^g, u_2^g, $\overline{d_3^b}$, $\overline{d_1^r}$, $\overline{u_3^g}$}, $\frac{e_3}{36} - \frac{i e_1}{36}$}	g[14,i ν]{*, $\frac{e_1}{36} + \frac{i e_3}{36}$}	g[14,ν*]{*, 0}	g[14,ν†]{{$\overline{d_1^r}$, $\overline{u_3^g}$, u_1^g, u_2^g, $\overline{d_3^b}$}, $-\frac{e_3}{36} + \frac{i e_1}{36}$}

ν_2

$\nu_2 = 3 e_7$

Λ[1,ν]{*, $-\frac{3 e_2}{2} - \frac{3 i e_6}{2}$}	Λ[1,i ν]{*, $\frac{3 e_6}{2} - \frac{3 i e_2}{2}$}	Λ[1,ν*]{*, $-\frac{3 e_2}{2} + \frac{3 i e_6}{2}$}	Λ[1,ν†]{*, $\frac{3 e_2}{2} + \frac{3 i e_6}{2}$}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,i ν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, $\frac{3 e_3}{2} - \frac{3 i e_1}{2}$}	Λ[5,i ν]{*, $\frac{3 e_1}{2} + \frac{3 i e_3}{2}$}	Λ[5,ν*]{*, $\frac{3 e_3}{2} + \frac{3 i e_1}{2}$}	Λ[5,ν†]{*, $-\frac{3 e_3}{2} + \frac{3 i e_1}{2}$}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,i ν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, $-\frac{e_2}{48} + \frac{i e_6}{48}$}	g[9,i ν]{{e_2, d_2^r}, $-\frac{e_6}{48} - \frac{i e_2}{48}$}	g[9,ν*]{*, $-\frac{e_2}{48} - \frac{i e_6}{48}$}	g[9,ν†]{*, $\frac{e_2}{48} - \frac{i e_6}{48}$}
g[10,ν]{{e_2, d_2^r}, $-\frac{e_6}{48} - \frac{i e_2}{48}$}	g[10,i ν]{*, $\frac{e_2}{48} - \frac{i e_6}{48}$}	g[10,ν*]{{$\overline{e_2}$, $\overline{d_2^r}$}, $-\frac{e_6}{48} + \frac{i e_2}{48}$}	g[10,ν†]{{e_2, d_2^r}, $\frac{e_6}{48} + \frac{i e_2}{48}$}
g[11,ν]{*, $\frac{e_1}{48} - \frac{i e_3}{48}$}	g[11,i ν]{{d_3^b, $\overline{u_1^g}$, $\overline{u_2^g}$, d_1^r, u_3^g}, $\frac{e_3}{48} + \frac{i e_1}{48}$}	g[11,ν*]{*, $\frac{e_1}{48} + \frac{i e_3}{48}$}	g[11,ν†]{*, $-\frac{e_1}{48} + \frac{i e_3}{48}$}
g[12,ν]{{d_1^r, u_3^g, d_3^b, $\overline{u_1^g}$, $\overline{u_2^g}$}, $-\frac{e_3}{48} - \frac{i e_1}{48}$}	g[12,i ν]{*, $\frac{e_1}{48} - \frac{i e_3}{48}$}	g[12,ν*]{{$\overline{d_1^r}$, $\overline{u_3^g}$, u_1^g, u_2^g, $\overline{d_3^b}$}, $-\frac{e_3}{48} + \frac{i e_1}{48}$}	g[12,ν†]{{d_3^b, $\overline{u_1^g}$, $\overline{u_2^g}$, d_1^r, u_3^g}, $\frac{e_3}{48} + \frac{i e_1}{48}$}
g[13,ν]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $\frac{e_4}{8} + \frac{i e_5}{8}$}	g[13,i ν]{{$\overline{d_2^b}$, $\overline{e_3}$}, $-\frac{e_5}{8} + \frac{i e_4}{8}$}	g[13,ν*]{{u_1^r, u_2^r}, $\frac{e_4}{8} - \frac{i e_5}{8}$}	g[13,ν†]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{8} - \frac{i e_5}{8}$}
g[14,ν]{{d_2^b, e_3}, $-\frac{e_5}{48} - \frac{i e_4}{48}$}	g[14,i ν]{{u_1^r, u_2^r}, $\frac{e_4}{48} - \frac{i e_5}{48}$}	g[14,ν*]{{$\overline{d_2^b}$, $\overline{e_3}$}, $-\frac{e_5}{48} + \frac{i e_4}{48}$}	g[14,ν†]{{e_3, d_2^b}, $\frac{e_5}{48} + \frac{i e_4}{48}$}

d_2^r

$d_2^r = 2 e_6 + 2 i e_2$

Λ[1,ν]{*, $2 i e_7$}	Λ[1,i ν]{{d_2^g, ν_1, $\overline{d_2^g}$}, $-2 e_7$}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, $-2 i e_7$}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, $e_2 - i e_6$}	Λ[3,i ν]{{e_2, d_2^r}, $e_6 + i e_2$}	Λ[3,ν*]{*, $e_2 - i e_6$}	Λ[3,ν†]{*, $-e_2 + i e_6$}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{{$\overline{e_3}$, $\overline{d_2^b}$}, $2 e_5 - 2 i e_4$}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, $e_2 - i e_6$}	Λ[8,i ν]{{e_2, d_2^r}, $e_6 + i e_2$}	Λ[8,ν*]{*, $e_2 - i e_6$}	Λ[8,ν†]{*, $-e_2 + i e_6$}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{*, $\frac{i e_7}{36}$}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{{ν_2, ν_1, d_2^g, $\overline{\nu_2}$, $\overline{d_2^g}$}, $\frac{e_7}{36}$}	g[10,ν†]{*, 0}
g[11,ν]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{36} - \frac{i e_5}{36}$}	g[11,i ν]{{$\overline{e_3}$, $\overline{d_2^b}$}, $\frac{e_5}{36} - \frac{i e_4}{36}$}	g[11,ν*]{*, 0}	g[11,ν†]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $\frac{e_4}{36} + \frac{i e_5}{36}$}
g[12,ν]{{$\overline{d_2^b}$, $\overline{e_3}$}, $-\frac{e_5}{36} + \frac{i e_4}{36}$}	g[12,i ν]{{$\overline{u_1^r}$, $\overline{u_2^r}$}, $-\frac{e_4}{36} - \frac{i e_5}{36}$}	g[12,ν*]{*, 0}	g[12,ν†]{{$\overline{e_3}$, $\overline{d_2^b}$}, $\frac{e_5}{36} - \frac{i e_4}{36}$}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, $\frac{e_1}{6} + \frac{i e_3}{6}$}	g[13,ν†]{*, 0}
g[14,ν]{{u_1^g, u_2^g, $\overline{d_3^b}$, $\overline{d_1^r}$, $\overline{u_3^g}$}, $\frac{e_3}{36} - \frac{i e_1}{36}$}	g[14,i ν]{*, $\frac{e_1}{36} + \frac{i e_3}{36}$}	g[14,ν*]{*, 0}	g[14,ν†]{{$\overline{d_1^r}$, $\overline{u_3^g}$, u_1^g, u_2^g, $\overline{d_3^b}$}, $-\frac{e_3}{36} + \frac{i e_1}{36}$}

d_2^g

$d_2^g = 2 e_7 + 2 i$

Λ[1,ν]{*, $-e_2 - i e_6$}	Λ[1,i ν]{{$\overline{e_2}$, $\overline{d_2^r}$}, $e_6 - i e_2$}	Λ[1,ν*]{*, $-e_2 + i e_6$}	Λ[1,ν†]{*, $e_2 + i e_6$}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,i ν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{{u_1^g, u_2^g, $\overline{d_3^b}$, $\overline{d_1^r}$, $\overline{u_3^g}$}, $e_3 - i e_1$}	Λ[5,i ν]{*, $e_1 + i e_3$}	Λ[5,ν*]{{d_3^b, $\overline{u_1^g}$, $\overline{u_2^g}$, d_1^r, u_3^g}, $e_3 + i e_1$}	Λ[5,ν†]{{$\overline{d_1^r}$, $\overline{u_3^g}$, u_1^g, u_2^g, $\overline{d_3^b}$}, $-e_3 + i e_1$}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,i ν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, $-\frac{e_2}{72} + \frac{i e_6}{72}$}	g[9,i ν]{{e_2, d_2^r}, $-\frac{e_6}{72} - \frac{i e_2}{72}$}	g[9,ν*]{*, $-\frac{e_2}{72} - \frac{i e_6}{72}$}	g[9,ν†]{*, $\frac{e_2}{72} - \frac{i e_6}{72}$}
g[10,ν]{{e_2, d_2^r}, $-\frac{e_6}{72} - \frac{i e_2}{72}$}	g[10,i ν]{*, $\frac{e_2}{72} - \frac{i e_6}{72}$}	g[10,ν*]{{$\overline{e_2}$, $\overline{d_2^r}$}, $-\frac{e_6}{72} + \frac{i e_2}{72}$}	g[10,ν†]{{e_2, d_2^r}, $\frac{e_6}{72} + \frac{i e_2}{72}$}
g[11,ν]{*, $\frac{e_1}{72} - \frac{i e_3}{72}$}	g[11,i ν]{{d_3^b, $\overline{u_1^g}$, $\overline{u_2^g}$, d_1^r, u_3^g}, $\frac{e_3}{72} + \frac{i e_1}{72}$}	g[11,ν*]{*, $\frac{e_1}{72} + \frac{i e_3}{72}$}	g[11,ν†]{*, $-\frac{e_1}{72} + \frac{i e_3}{72}$}
g[12,ν]{{d_1^r, u_3^g, d_3^b, $\overline{u_1^g}$, $\overline{u_2^g}$}, $-\frac{e_3}{72} - \frac{i e_1}{72}$}	g[12,i ν]{*, $\frac{e_1}{72} - \frac{i e_3}{72}$}	g[12,ν*]{{$\overline{d_1^r}$, $\overline{u_3^g}$, u_1^g, u_2^g, $\overline{d_3^b}$}, $-\frac{e_3}{72} + \frac{i e_1}{72}$}	g[12,ν†]{{d_3^b, $\overline{u_1^g}$, $\overline{u_2^g}$, d_1^r, u_3^g}, $\frac{e_3}{72} + \frac{i e_1}{72}$}

d_2^b

$d_2^b = -2 e_5 - 2 i e_4$

Λ[1,ν]{*, 0}	Λ[1,i ν]{*, 0}	Λ[1,ν*]{*, $-2 e_1 - 2 i e_3$}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{{u_1^r, u_2^r}, $-e_4 + i e_5$}	Λ[3,i ν]{{$\overline{d_2^b}$, e_3}, $-e_5 - i e_4$}	Λ[3,ν*]{{u_1^r, u_2^r}, $-e_4 + i e_5$}	Λ[3,ν†]{{u_1^r, u_2^r}, $e_4 - i e_5$}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{{$\overline{e_2}$, $\overline{d_2^r}$}, $2 e_6 - 2 i e_2$}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{{u_1^r, u_2^r}, $-e_4 + i e_5$}	Λ[8,i ν]{{$\overline{d_2^b}$, e_3}, $-e_5 - i e_4$}	Λ[8,ν*]{{u_1^r, u_2^r}, $-e_4 + i e_5$}	Λ[8,ν†]{{u_1^r, u_2^r}, $e_4 - i e_5$}
g[9,ν]{*, $-\frac{e_1}{36} - \frac{i e_3}{36}$}	g[9,i ν]{{u_1^g, u_2^g, $\overline{d_3^b}$, $\overline{d_1^r}$, $\overline{u_3^g}$}, $\frac{e_3}{36} - \frac{i e_1}{36}$}	g[9,ν*]{*, 0}	g[9,ν†]{*, $\frac{e_1}{36} + \frac{i e_3}{36}$}
g[10,ν]{{u_1^g, u_2^g, $\overline{d_3^b}$, $\overline{d_1^r}$, $\overline{u_3^g}$}, $\frac{e_3}{36} - \frac{i e_1}{36}$}	g[10,i ν]{*, $\frac{e_1}{36} + \frac{i e_3}{36}$}	g[10,ν*]{*, 0}	g[10,ν†]{{$\overline{d_1^r}$, $\overline{u_3^g}$, u_1^g, u_2^g, $\overline{d_3^b}$}, $-\frac{e_3}{36} + \frac{i e_1}{36}$}
g[11,ν]{*, $-\frac{e_2}{36} - \frac{i e_6}{36}$}	g[11,i ν]{{$\overline{e_2}$, $\overline{d_2^r}$}, $\frac{e_6}{36} - \frac{i e_2}{36}$}	g[11,ν*]{*, 0}	g[11,ν†]{*, $\frac{e_2}{36} + \frac{i e_6}{36}$}
g[12,ν]{{$\overline{e_2}$, $\overline{d_2^r}$}, $-\frac{e_6}{36} + \frac{i e_2}{36}$}	g[12,i ν]{*, $-\frac{e_2}{36} - \frac{i e_6}{36}$}	g[12,ν*]{*, 0}	g[12,ν†]{{$\overline{e_2}$, $\overline{d_2^r}$}, $\frac{e_6}{36} - \frac{i e_2}{36}$}

g[13,v]{{ū1^r, ū2^r}, e4/12 + i e5/12}	g[13,i v]{{d̄2^b, ē3}, −e5/12 + i e4/12}	g[13,v*]{{u1^r, u2^r}, e4/12 − i e5/12}	g[13,v†]{{ū1^r, ū2^r}, −e4/12 − i e5/12}
g[14,v]{{d2^b, e3}, −e5/72 − i e4/72}	g[14,i v]{{u1^r, u2^r}, e4/72 − i e5/72}	g[14,v*]{{d̄2^b, ē3}, −e5/72 + i e4/72}	g[14,v†]{{e3, d2^b}, e5/72 + i e4/72}

g[13,v]{*, i e7/6}	g[13,i v]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7/6}	g[13,v*]{*, 0}	g[13,v†]{*, −i e7/6}
g[14,v]{*, 0}	g[14,i v]{*, 0}	g[14,v*]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7/36}	g[14,v†]{*, 0}

u2^r

u2^r = −e4 + i e5

Λ[1,v]{*, 0}	Λ[1,i v]{*, 0}	Λ[1,v*]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3 + i e1}	Λ[1,v†]{*, 0}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{{e3, d2^b}, e5/2 + i e4/2}	Λ[3,i v]{{u1^r, u2^r}, −e4/2 + i e5/2}	Λ[3,v*]{{e3, d2^b}, e5/2 + i e4/2}	Λ[3,v†]{{d̄2^b, ē3}, −e5/2 − i e4/2}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{*, 0}	Λ[5,i v]{*, 0}	Λ[5,v*]{*, −e2 − i e6}	Λ[5,v†]{*, 0}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{{e3, d2^b}, e5/2 + i e4/2}	Λ[8,i v]{{u1^r, u2^r}, −e4/2 + i e5/2}	Λ[8,v*]{{e3, d2^b}, e5/2 + i e4/2}	Λ[8,v†]{{d̄2^b, ē3}, −e5/2 − i e4/2}
g[9,v]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3/72 + i e1/72}	g[9,i v]{*, −e1/72 − i e3/72}	g[9,v*]{*, 0}	g[9,v†]{{u1^g, u2^g, d̄3^b, d̄1^r, ū3^g}, e3/72 − i e1/72}
g[10,v]{*, −e1/72 − i e3/72}	g[10,i v]{{u1^g, u2^g, d̄3^b, d̄1^r, ū3^g}, e3/72 − i e1/72}	g[10,v*]{*, 0}	g[10,v†]{*, e1/72 + i e3/72}
g[11,v]{{ē2, d̄2^r}, −e6/72 + i e2/72}	g[11,i v]{*, −e2/72 − i e6/72}	g[11,v*]{*, 0}	g[11,v†]{{ē2, d̄2^r}, e6/72 − i e2/72}
g[12,v]{*, e2/72 + i e6/72}	g[12,i v]{{ē2, d̄2^r}, −e6/72 + i e2/72}	g[12,v*]{*, 0}	g[12,v†]{*, −e2/72 − i e6/72}
g[13,v]{{v2, v1, d2^g, v̄2, d̄2^g}, e7/12}	g[13,i v]{*, i e7/12}	g[13,v*]{*, 0}	g[13,v†]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7/12}
g[14,v]{*, 0}	g[14,i v]{*, 0}	g[14,v*]{*, i e7/72}	g[14,v†]{*, 0}

u2^g

u2^g = e3 − i e1

Λ[1,v]{{u1^r, u2^r}, −e4 + i e5}	Λ[1,i v]{{d2^b, e3}, −e5 − i e4}	Λ[1,v*]{*, 0}	Λ[1,v†]{{u1^r, u2^r}, e4 − i e5}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{*, e1/2 + i e3/2}	Λ[3,i v]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3/2 + i e1/2}	Λ[3,v*]{*, e1/2 + i e3/2}	Λ[3,v†]{*, −e1/2 − i e3/2}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{*, 0}	Λ[5,i v]{*, 0}	Λ[5,v*]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7}	Λ[5,v†]{*, 0}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{*, e1/2 + i e3/2}	Λ[8,i v]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3/2 + i e1/2}	Λ[8,v*]{*, e1/2 + i e3/2}	Λ[8,v†]{*, −e1/2 − i e3/2}
g[9,v]{*, 0}	g[9,i v]{*, 0}	g[9,v*]{{u1^r, u2^r}, −e4/72 + i e5/72}	g[9,v†]{*, 0}
g[10,v]{*, 0}	g[10,i v]{*, 0}	g[10,v*]{{e3, d2^b}, e5/72 + i e4/72}	g[10,v†]{*, 0}
g[11,v]{*, i e7/72}	g[11,i v]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7/72}	g[11,v*]{*, 0}	g[11,v†]{*, −i e7/72}
g[12,v]{{v2, v1, d2^g, v̄2, d̄2^g}, e7/72}	g[12,i v]{*, i e7/72}	g[12,v*]{*, 0}	g[12,v†]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7/72}
g[13,v]{*, −e2/12 + i e6/12}	g[13,i v]{{e2, d2^r}, −e6/12 − i e2/12}	g[13,v*]{*, 0}	g[13,v†]{*, e2/12 − i e6/12}
g[14,v]{*, 0}	g[14,i v]{*, 0}	g[14,v*]{{e2, d2^r}, −e6/72 − i e2/72}	g[14,v†]{*, 0}

u2^b

u2^b = −e2/2

Λ[1,v]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7/4}	Λ[1,i v]{*, −i e7/4}	Λ[1,v*]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7/4}	Λ[1,v†]{{v2, v1, d2^g, v̄2, d̄2^g}, e7/4}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{{e2, v1, d2^r, ē2, d̄2^r}, e6/4}	Λ[3,i v]{*, i e6/4}	Λ[3,v*]{{e2, v1, d2^r, ē2, d̄2^r}, e6/4}	Λ[3,v†]{{e2, v1, d2^r, ē2, d̄2^r}, −e6/4}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{{u1^r, u2^r}, e4/4 − i e5/4}	Λ[5,i v]{{e3, d2^b}, e5/4 + i e4/4}	Λ[5,v*]{{ū1^r, ū2^r}, e4/4 + i e5/4}	Λ[5,v†]{{u1^r, u2^r}, −e4/4 + i e5/4}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{{e2, v1, d2^r, ē2, d̄2^r}, e6/4}	Λ[8,i v]{*, i e6/4}	Λ[8,v*]{{e2, v1, d2^r, ē2, d̄2^r}, e6/4}	Λ[8,v†]{{e2, v1, d2^r, ē2, d̄2^r}, −e6/4}
g[9,v]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7/288}	g[9,i v]{*, −i e7/288}	g[9,v*]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7/288}	g[9,v†]{{v2, v1, d2^g, v̄2, d̄2^g}, e7/288}
g[10,v]{*, −i e7/288}	g[10,i v]{{v2, v1, d2^g, v̄2, d̄2^g}, e7/288}	g[10,v*]{*, i e7/288}	g[10,v†]{*, i e7/288}
g[11,v]{{ē3, d̄2^b}, e5/288 − i e4/288}	g[11,i v]{{ū1^r, ū2^r}, e4/288 + i e5/288}	g[11,v*]{{e3, d2^b}, e5/288 + i e4/288}	g[11,v†]{{d̄2^b, ē3}, −e5/288 + i e4/288}
g[12,v]{{ū1^r, ū2^r}, −e4/288 − i e5/288}	g[12,i v]{{ē3, d̄2^b}, e5/288 − i e4/288}	g[12,v*]{{ū1^r, ū2^r}, −e4/288 + i e5/288}	g[12,v†]{{ū1^r, ū2^r}, e4/288 + i e5/288}
g[13,v]{{d̄1^r, ū3^g, d̄3^b, ū1^g, ū2^g}, −e3/48 − i e1/48}	g[13,i v]{*, e1/48 − i e3/48}	g[13,v*]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3/48 + i e1/48}	g[13,v†]{{d̄3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/48 + i e1/48}
g[14,v]{*, e1/288 + i e3/288}	g[14,i v]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3/288 + i e1/288}	g[14,v*]{*, e1/288 − i e3/288}	g[14,v†]{*, −e1/288 − i e3/288}

e3

e3 = 2 e5 + 2 i e4

Λ[1,v]{*, 0}	Λ[1,i v]{*, 0}	Λ[1,v*]{*, 2 e1 + 2 i e3}	Λ[1,v†]{*, 0}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{{u1^r, u2^r}, e4 − i e5}	Λ[3,i v]{{e3, d2^b}, e5 + i e4}	Λ[3,v*]{{u1^r, u2^r}, e4 − i e5}	Λ[3,v†]{{u1^r, u2^r}, −e4 + i e5}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{*, 0}	Λ[5,i v]{*, 0}	Λ[5,v*]{{e2, d2^r}, −2 e6 + 2 i e2}	Λ[5,v†]{*, 0}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{{u1^r, u2^r}, e4 − i e5}	Λ[8,i v]{{e3, d2^b}, e5 + i e4}	Λ[8,v*]{{u1^r, u2^r}, e4 − i e5}	Λ[8,v†]{{u1^r, u2^r}, −e4 + i e5}
g[9,v]{*, e1/36 + i e3/36}	g[9,i v]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3/36 + i e1/36}	g[9,v*]{*, 0}	g[9,v†]{*, −e1/36 − i e3/36}
g[10,v]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3/36 + i e1/36}	g[10,i v]{*, −e1/36 − i e3/36}	g[10,v*]{*, 0}	g[10,v†]{{u1^g, u2^g, d̄3^b, d̄1^r, ū3^g}, e3/36 − i e1/36}
g[11,v]{*, e2/36 + i e6/36}	g[11,i v]{{ē2, d̄2^r}, −e6/36 + i e2/36}	g[11,v*]{*, 0}	g[11,v†]{*, −e2/36 − i e6/36}
g[12,v]{{ē2, d̄2^r}, e6/36 − i e2/36}	g[12,i v]{*, e2/36 + i e6/36}	g[12,v*]{*, 0}	g[12,v†]{{ē2, d̄2^r}, −e6/36 + i e2/36}
g[13,v]{*, −i e7/6}	g[13,i v]{{v2, v1, d2^g, v̄2, d̄2^g}, e7/6}	g[13,v*]{*, 0}	g[13,v†]{*, i e7/6}
g[14,v]{*, 0}	g[14,i v]{*, 0}	g[14,v*]{{v2, v1, d2^g, v̄2, d̄2^g}, e7/36}	g[14,v†]{*, 0}

v3

v3 = 2 i

Λ[1,v]{*, 0}	Λ[1,i v]{*, 0}	Λ[1,v*]{*, 0}	Λ[1,v†]{*, 0}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{*, 0}	Λ[3,i v]{*, 0}	Λ[3,v*]{*, 0}	Λ[3,v†]{*, 0}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{*, 0}	Λ[5,i v]{*, 0}	Λ[5,v*]{*, 0}	Λ[5,v†]{*, 0}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{*, 0}	Λ[8,i v]{*, 0}	Λ[8,v*]{*, 0}	Λ[8,v†]{*, 0}
g[9,v]{*, 0}	g[9,i v]{*, 0}	g[9,v*]{*, 0}	g[9,v†]{*, 0}
g[10,v]{*, 0}	g[10,i v]{*, 0}	g[10,v*]{*, 0}	g[10,v†]{*, 0}
g[11,v]{*, 0}	g[11,i v]{*, 0}	g[11,v*]{*, 0}	g[11,v†]{*, 0}
g[12,v]{*, 0}	g[12,i v]{*, 0}	g[12,v*]{*, 0}	g[12,v†]{*, 0}
g[13,v]{*, 0}	g[13,i v]{*, 0}	g[13,v*]{*, 0}	g[13,v†]{*, 0}
g[14,v]{*, 0}	g[14,i v]{*, 0}	g[14,v*]{*, 0}	g[14,v†]{*, 0}

d3^r

d3^r = 0

Λ[1,v]{*, 0}	Λ[1,i v]{*, 0}	Λ[1,v*]{*, 0}	Λ[1,v†]{*, 0}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{*, 0}	Λ[3,i v]{*, 0}	Λ[3,v*]{*, 0}	Λ[3,v†]{*, 0}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{*, 0}	Λ[5,i v]{*, 0}	Λ[5,v*]{*, 0}	Λ[5,v†]{*, 0}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{*, 0}	Λ[8,i v]{*, 0}	Λ[8,v*]{*, 0}	Λ[8,v†]{*, 0}
g[9,v]{*, 0}	g[9,i v]{*, 0}	g[9,v*]{*, 0}	g[9,v†]{*, 0}
g[10,v]{*, 0}	g[10,i v]{*, 0}	g[10,v*]{*, 0}	g[10,v†]{*, 0}
g[11,v]{*, 0}	g[11,i v]{*, 0}	g[11,v*]{*, 0}	g[11,v†]{*, 0}
g[12,v]{*, 0}	g[12,i v]{*, 0}	g[12,v*]{*, 0}	g[12,v†]{*, 0}
g[13,v]{*, 0}	g[13,i v]{*, 0}	g[13,v*]{*, 0}	g[13,v†]{*, 0}
g[14,v]{*, 0}	g[14,i v]{*, 0}	g[14,v*]{*, 0}	g[14,v†]{*, 0}

d3^g

d3^g = 0

Λ[1,v]{*, 0}	Λ[1,i v]{*, 0}	Λ[1,v*]{*, 0}	Λ[1,v†]{*, 0}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{*, 0}	Λ[3,i v]{*, 0}	Λ[3,v*]{*, 0}	Λ[3,v†]{*, 0}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{*, 0}	Λ[5,i v]{*, 0}	Λ[5,v*]{*, 0}	Λ[5,v†]{*, 0}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{*, 0}	Λ[8,i v]{*, 0}	Λ[8,v*]{*, 0}	Λ[8,v†]{*, 0}
g[9,v]{*, 0}	g[9,i v]{*, 0}	g[9,v*]{*, 0}	g[9,v†]{*, 0}
g[10,v]{*, 0}	g[10,i v]{*, 0}	g[10,v*]{*, 0}	g[10,v†]{*, 0}
g[11,v]{*, 0}	g[11,i v]{*, 0}	g[11,v*]{*, 0}	g[11,v†]{*, 0}
g[12,v]{*, 0}	g[12,i v]{*, 0}	g[12,v*]{*, 0}	g[12,v†]{*, 0}
g[13,v]{*, 0}	g[13,i v]{*, 0}	g[13,v*]{*, 0}	g[13,v†]{*, 0}
g[14,v]{*, 0}	g[14,i v]{*, 0}	g[14,v*]{*, 0}	g[14,v†]{*, 0}

d3^b

d3^b = e3 + i e1

Λ[1,v]{*, 0}	Λ[1,i v]{*, 0}	Λ[1,v*]{{ū1^r, ū2^r}, −e4 − i e5}	Λ[1,v†]{*, 0}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{*, e1/2 − i e3/2}	Λ[3,i v]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, e3/2 + i e1/2}	Λ[3,v*]{*, e1/2 − i e3/2}	Λ[3,v†]{*, −e1/2 + i e3/2}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7}	Λ[5,i v]{*, −i e7}	Λ[5,v*]{*, 0}	Λ[5,v†]{{v2, v1, d2^g, v̄2, d̄2^g}, e7}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{*, e1/2 − i e3/2}	Λ[8,i v]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, e3/2 + i e1/2}	Λ[8,v*]{*, e1/2 − i e3/2}	Λ[8,v†]{*, −e1/2 + i e3/2}
g[9,v]{{ū1^r, ū2^r}, −e4/72 − i e5/72}	g[9,i v]{{e3, d2^b}, e5/72 − i e4/72}	g[9,v*]{*, 0}	g[9,v†]{{ū1^r, ū2^r}, e4/72 + i e5/72}
g[10,v]{{e3, d2^b}, e5/72 − i e4/72}	g[10,i v]{{ū1^r, ū2^r}, e4/72 + i e5/72}	g[10,v*]{*, 0}	g[10,v†]{{d̄2^b, ē3}, −e5/72 + i e4/72}
g[11,v]{*, 0}	g[11,i v]{*, 0}	g[11,v*]{*, −i e7/72}	g[11,v†]{*, 0}
g[12,v]{*, 0}	g[12,i v]{*, 0}	g[12,v*]{{v2, v1, d2^g, v̄2, d̄2^g}, e7/72}	g[12,v†]{*, 0}
g[13,v]{*, 0}	g[13,i v]{*, 0}	g[13,v*]{*, −e2/12 − i e6/12}	g[13,v†]{*, 0}
g[14,v]{{ē2, d̄2^r}, −e6/72 + i e2/72}	g[14,i v]{*, −e2/72 − i e6/72}	g[14,v*]{*, 0}	g[14,v†]{{ē2, d̄2^r}, e6/72 − i e2/72}

u3^r

u3^r = 0

Λ[1,v]{*, 0}	Λ[1,i v]{*, 0}	Λ[1,v*]{*, 0}	Λ[1,v†]{*, 0}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{*, 0}	Λ[3,i v]{*, 0}	Λ[3,v*]{*, 0}	Λ[3,v†]{*, 0}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{*, 0}	Λ[5,i v]{*, 0}	Λ[5,v*]{*, 0}	Λ[5,v†]{*, 0}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{*, 0}	Λ[8,i v]{*, 0}	Λ[8,v*]{*, 0}	Λ[8,v†]{*, 0}
g[9,v]{*, 0}	g[9,i v]{*, 0}	g[9,v*]{*, 0}	g[9,v†]{*, 0}
g[10,v]{*, 0}	g[10,i v]{*, 0}	g[10,v*]{*, 0}	g[10,v†]{*, 0}
g[11,v]{*, 0}	g[11,i v]{*, 0}	g[11,v*]{*, 0}	g[11,v†]{*, 0}
g[12,v]{*, 0}	g[12,i v]{*, 0}	g[12,v*]{*, 0}	g[12,v†]{*, 0}
g[13,v]{*, 0}	g[13,i v]{*, 0}	g[13,v*]{*, 0}	g[13,v†]{*, 0}
g[14,v]{*, 0}	g[14,i v]{*, 0}	g[14,v*]{*, 0}	g[14,v†]{*, 0}

u3^g

u3^g = −e3 − i e1

Λ[1,v]{*, 0}	Λ[1,i v]{*, 0}	Λ[1,v*]{{ū1^r, ū2^r}, e4 + i e5}	Λ[1,v†]{*, 0}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{*, −e1/2 + i e3/2}	Λ[3,i v]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3/2 − i e1/2}	Λ[3,v*]{*, −e1/2 + i e3/2}	Λ[3,v†]{*, e1/2 − i e3/2}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{{v2, v1, d2^g, v̄2, d̄2^g}, e7}	Λ[5,i v]{*, i e7}	Λ[5,v*]{*, 0}	Λ[5,v†]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{*, −e1/2 + i e3/2}	Λ[8,i v]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3/2 − i e1/2}	Λ[8,v*]{*, −e1/2 + i e3/2}	Λ[8,v†]{*, e1/2 − i e3/2}
g[9,v]{{ū1^r, ū2^r}, e4/72 + i e5/72}	g[9,i v]{{d̄2^b, ē3}, −e5/72 + i e4/72}	g[9,v*]{*, 0}	g[9,v†]{{ū1^r, ū2^r}, −e4/72 − i e5/72}
g[10,v]{{d̄2^b, ē3}, −e5/72 + i e4/72}	g[10,i v]{{ū1^r, ū2^r}, −e4/72 − i e5/72}	g[10,v*]{*, 0}	g[10,v†]{{d2^b, e3}, e5/72 − i e4/72}
g[11,v]{*, 0}	g[11,i v]{*, 0}	g[11,v*]{*, i e7/72}	g[11,v†]{*, 0}
g[12,v]{*, 0}	g[12,i v]{*, 0}	g[12,v*]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7/72}	g[12,v†]{*, 0}
g[13,v]{*, 0}	g[13,i v]{*, 0}	g[13,v*]{*, e2/12 + i e6/12}	g[13,v†]{*, 0}
g[14,v]{{ē2, d̄2^r}, e6/72 − i e2/72}	g[14,i v]{*, e2/72 + i e6/72}	g[14,v*]{*, 0}	g[14,v†]{{ē2, d̄2^r}, −e6/72 + i e2/72}

u3^b

u3^b = 0

Λ[1,v]{*, 0}	Λ[1,i v]{*, 0}	Λ[1,v*]{*, 0}	Λ[1,v†]{*, 0}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{*, 0}	Λ[3,i v]{*, 0}	Λ[3,v*]{*, 0}	Λ[3,v†]{*, 0}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{*, 0}	Λ[5,i v]{*, 0}	Λ[5,v*]{*, 0}	Λ[5,v†]{*, 0}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{*, 0}	Λ[8,i v]{*, 0}	Λ[8,v*]{*, 0}	Λ[8,v†]{*, 0}
g[9,v]{*, 0}	g[9,i v]{*, 0}	g[9,v*]{*, 0}	g[9,v†]{*, 0}
g[10,v]{*, 0}	g[10,i v]{*, 0}	g[10,v*]{*, 0}	g[10,v†]{*, 0}
g[11,v]{*, 0}	g[11,i v]{*, 0}	g[11,v*]{*, 0}	g[11,v†]{*, 0}
g[12,v]{*, 0}	g[12,i v]{*, 0}	g[12,v*]{*, 0}	g[12,v†]{*, 0}
g[13,v]{*, 0}	g[13,i v]{*, 0}	g[13,v*]{*, 0}	g[13,v†]{*, 0}
g[14,v]{*, 0}	g[14,i v]{*, 0}	g[14,v*]{*, 0}	g[14,v†]{*, 0}

ē1

ē1 = −3 e3 + i e1

Λ[1,v]{*, 2 e4 − 2 i e5}	Λ[1,i v]{{e3, d2^b}, 2 e5 + 2 i e4}	Λ[1,v*]{{ū1^r, ū2^r}, e4 + i e5}	Λ[1,v†]{*, −2 e4 + 2 i e5}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{*, −5 e1/2 − i e3/2}	Λ[3,i v]{*, e3/2 − 3 i e1/2}	Λ[3,v*]{*, −3 e1/2 − i e3/2}	Λ[3,v†]{*, 5 e1/2 + i e3/2}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{{v2, v1, d2^g, v̄2, d̄2^g}, e7}	Λ[5,i v]{*, i e7}	Λ[5,v*]{{v2, v1, d2^g}, 2 e7}	Λ[5,v†]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{*, −5 e1/2 − i e3/2}	Λ[8,i v]{*, e3/2 − 3 i e1/2}	Λ[8,v*]{*, −3 e1/2 − i e3/2}	Λ[8,v†]{*, 5 e1/2 + i e3/2}
g[9,v]{{ū1^r, ū2^r}, e4/72 + i e5/72}	g[9,i v]{{d̄2^b, ē3}, −e5/72 + i e4/72}	g[9,v*]{{ū1^r, ū2^r}, e4/36 − i e5/36}	g[9,v†]{{ū1^r, ū2^r}, −e4/72 − i e5/72}
g[10,v]{{d̄2^b, ē3}, −e5/72 + i e4/72}	g[10,i v]{{ū1^r, ū2^r}, −e4/72 − i e5/72}	g[10,v*]{{d̄2^b, ē3}, −e5/36 − i e4/36}	g[10,v†]{{e3, d2^b}, e5/72 − i e4/72}
g[11,v]{*, −i e7/36}	g[11,i v]{{v2, v1, d2^g, v̄2, d̄2^g}, e7/36}	g[11,v*]{*, i e7/72}	g[11,v†]{*, i e7/36}
g[12,v]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7/36}	g[12,i v]{*, −i e7/36}	g[12,v*]{{v2, v1, d2^g, v̄2, d̄2^g}, −e7/72}	g[12,v†]{{v2, v1, d2^g, v̄2, d̄2^g}, e7/36}
g[13,v]{*, e2/6 − i e6/6}	g[13,i v]{{e2, d2^r}, e6/6 + i e2/6}	g[13,v*]{*, e2/12 + i e6/12}	g[13,v†]{*, −e2/6 + i e6/6}
g[14,v]{{ē2, d̄2^r}, e6/72 − i e2/72}	g[14,i v]{*, e2/72 + i e6/72}	g[14,v*]{{e2, d2^r}, e6/36 + i e2/36}	g[14,v†]{{ē2, d̄2^r}, −e6/72 + i e2/72}

v̄1

v̄1 = 1. + 0. i

Λ[1,v]{*, 0}	Λ[1,i v]{*, 0}	Λ[1,v*]{*, 0}	Λ[1,v†]{*, 0}
Λ[2,v]{*, 0}	Λ[2,i v]{*, 0}	Λ[2,v*]{*, 0}	Λ[2,v†]{*, 0}
Λ[3,v]{*, 0}	Λ[3,i v]{*, 0}	Λ[3,v*]{*, 0}	Λ[3,v†]{*, 0}
Λ[4,v]{*, 0}	Λ[4,i v]{*, 0}	Λ[4,v*]{*, 0}	Λ[4,v†]{*, 0}
Λ[5,v]{*, 0}	Λ[5,i v]{*, 0}	Λ[5,v*]{*, 0}	Λ[5,v†]{*, 0}
Λ[6,v]{*, 0}	Λ[6,i v]{*, 0}	Λ[6,v*]{*, 0}	Λ[6,v†]{*, 0}
Λ[7,v]{*, 0}	Λ[7,i v]{*, 0}	Λ[7,v*]{*, 0}	Λ[7,v†]{*, 0}
Λ[8,v]{*, 0}	Λ[8,i v]{*, 0}	Λ[8,v*]{*, 0}	Λ[8,v†]{*, 0}

g[9,ν]{*, 0}	g[9,iν]{*, 0}	g[9,ν*]{*, 0}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,iν]{*, 0}	g[10,ν*]{*, 0}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,iν]{*, 0}	g[11,ν*]{*, 0}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,iν]{*, 0}	g[12,ν*]{*, 0}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,iν]{*, 0}	g[13,ν*]{*, 0}	g[13,ν†]{*, 0}
g[14,ν]{*, 0}	g[14,iν]{*, 0}	g[14,ν*]{*, 0}	g[14,ν†]{*, 0}

d̄1^r

$\overline{d_1^r} = -e_3 + i e_1$

Λ[1,ν]{{u1^r, u2^r}, e4 − i e5}	Λ[1,iν]{{e3, d2^b}, e5 + i e4}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{{u1^r, u2^r}, −e4 + i e5}
Λ[2,ν]{*, 0}	Λ[2,iν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, −e1/2 − i e3/2}	Λ[3,iν]{{u1^g, u2^g, d̄3^b, d̄1^r, ū3^g}, e3/2 − i e1/2}	Λ[3,ν*]{*, −e1/2 − i e3/2}	Λ[3,ν†]{*, e1/2 + i e3/2}
Λ[4,ν]{*, 0}	Λ[4,iν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,iν]{*, 0}	Λ[5,ν*]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,iν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,iν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, −e1/2 − i e3/2}	Λ[8,iν]{{u1^g, u2^g, d̄3^b, d̄1^r, ū3^g}, e3/2 − i e1/2}	Λ[8,ν*]{*, −e1/2 − i e3/2}	Λ[8,ν†]{*, e1/2 + i e3/2}
g[9,ν]{*, 0}	g[9,iν]{*, 0}	g[9,ν*]{{u1^r, u2^r}, e4/72 − i e5/72}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,iν]{*, 0}	g[10,ν*]{{d2^b, e3}, −e5/72 − i e4/72}	g[10,ν†]{*, 0}
g[11,ν]{*, −i e7/72}	g[11,iν]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/72}	g[11,ν*]{*, 0}	g[11,ν†]{*, i e7/72}
g[12,ν]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, −e7/72}	g[12,iν]{*, −i e7/72}	g[12,ν*]{*, 0}	g[12,ν†]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/72}
g[13,ν]{*, e2/12 − i e6/12}	g[13,iν]{{e2, d2^r}, e6/12 + i e2/12}	g[13,ν*]{*, 0}	g[13,ν†]{*, −e2/12 + i e6/12}
g[14,ν]{*, 0}	g[14,iν]{*, 0}	g[14,ν*]{{e2, d2^r}, e6/72 + i e2/72}	g[14,ν†]{*, 0}

d̄1^g

$\overline{d_1^g} = 0$

Λ[1,ν]{*, 0}	Λ[1,iν]{*, 0}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,iν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,iν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,iν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,iν]{*, 0}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,iν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,iν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,iν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, 0}	g[9,iν]{*, 0}	g[9,ν*]{*, 0}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,iν]{*, 0}	g[10,ν*]{*, 0}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,iν]{*, 0}	g[11,ν*]{*, 0}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,iν]{*, 0}	g[12,ν*]{*, 0}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,iν]{*, 0}	g[13,ν*]{*, 0}	g[13,ν†]{*, 0}
g[14,ν]{*, 0}	g[14,iν]{*, 0}	g[14,ν*]{*, 0}	g[14,ν†]{*, 0}

d̄1^b

$\overline{d_1^b} = 0$

Λ[1,ν]{*, 0}	Λ[1,iν]{*, 0}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,iν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,iν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,iν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,iν]{*, 0}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,iν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,iν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,iν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, 0}	g[9,iν]{*, 0}	g[9,ν*]{*, 0}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,iν]{*, 0}	g[10,ν*]{*, 0}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,iν]{*, 0}	g[11,ν*]{*, 0}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,iν]{*, 0}	g[12,ν*]{*, 0}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,iν]{*, 0}	g[13,ν*]{*, 0}	g[13,ν†]{*, 0}
g[14,ν]{*, 0}	g[14,iν]{*, 0}	g[14,ν*]{*, 0}	g[14,ν†]{*, 0}

ū1^r

$\overline{u_1^r} = 2\left(-\frac{e_4}{4} - \frac{i e_5}{4}\right) - 2\left(\frac{e_4}{4} + \frac{i e_5}{4}\right)$

Λ[1,ν]{{d̄1^r, ū3^g, d3^b, ū1^g, ū2^g}, −e3 − i e1}	Λ[1,iν]{*, e1 − i e3}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3 + i e1}
Λ[2,ν]{*, 0}	Λ[2,iν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{{ē3, d̄2^b}, e5/2 − i e4/2}	Λ[3,iν]{{ū1^r, ū2^r}, e4/2 + i e5/2}	Λ[3,ν*]{{ē3, d̄2^b}, e5/2 − i e4/2}	Λ[3,ν†]{{d̄2^b, ē3}, −e5/2 + i e4/2}
Λ[4,ν]{*, 0}	Λ[4,iν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, −e2 + i e6}	Λ[5,iν]{{e2, d2^r}, −e6 − i e2}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{*, e2 − i e6}
Λ[6,ν]{*, 0}	Λ[6,iν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,iν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{{ē3, d̄2^b}, e5/2 − i e4/2}	Λ[8,iν]{{ū1^r, ū2^r}, e4/2 + i e5/2}	Λ[8,ν*]{{ē3, d̄2^b}, e5/2 − i e4/2}	Λ[8,ν†]{{d̄2^b, ē3}, −e5/2 + i e4/2}
g[9,ν]{*, 0}	g[9,iν]{*, 0}	g[9,ν*]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, −e3/72 − i e1/72}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,iν]{*, 0}	g[10,ν*]{*, −e1/72 + i e3/72}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,iν]{*, 0}	g[11,ν*]{{e2, d2^r}, −e6/72 − i e2/72}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,iν]{*, 0}	g[12,ν*]{*, −e2/72 − i e6/72}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,iν]{*, 0}	g[13,ν*]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/12}	g[13,ν†]{*, 0}
g[14,ν]{*, −i e7/72}	g[14,iν]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/72}	g[14,ν*]{*, 0}	g[14,ν†]{*, i e7/72}

ū1^g

$\overline{u_1^g} = 4\left(\frac{e_3}{4} + \frac{i e_1}{4}\right)$

Λ[1,ν]{*, 0}	Λ[1,iν]{*, 0}	Λ[1,ν*]{{ū1^r, ū2^r}, −e4 − i e5}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,iν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, e1/2 − i e3/2}	Λ[3,iν]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/2 + i e1/2}	Λ[3,ν*]{*, e1/2 − i e3/2}	Λ[3,ν†]{*, −e1/2 + i e3/2}
Λ[4,ν]{*, 0}	Λ[4,iν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, −e7}	Λ[5,iν]{*, −i e7}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7}
Λ[6,ν]{*, 0}	Λ[6,iν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,iν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, e1/2 − i e3/2}	Λ[8,iν]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/2 + i e1/2}	Λ[8,ν*]{*, e1/2 − i e3/2}	Λ[8,ν†]{*, −e1/2 + i e3/2}
g[9,ν]{{ū1^r, ū2^r}, −e4/72 − i e5/72}	g[9,iν]{{ē3, d̄2^b}, e5/72 − i e4/72}	g[9,ν*]{*, 0}	g[9,ν†]{{ū1^r, ū2^r}, e4/72 + i e5/72}
g[10,ν]{{ē3, d̄2^b}, e5/72 − i e4/72}	g[10,iν]{{ū1^r, ū2^r}, e4/72 + i e5/72}	g[10,ν*]{*, 0}	g[10,ν†]{{d̄2^b, ē3}, −e5/72 + i e4/72}
g[11,ν]{*, 0}	g[11,iν]{*, 0}	g[11,ν*]{*, −i e7/72}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,iν]{*, 0}	g[12,ν*]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/72}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,iν]{*, 0}	g[13,ν*]{*, −e2/12 − i e6/12}	g[13,ν†]{*, 0}
g[14,ν]{{e2, d2^r}, −e6/72 + i e2/72}	g[14,iν]{*, −e2/72 − i e6/72}	g[14,ν*]{*, 0}	g[14,ν†]{{e2, d2^r}, e6/72 − i e2/72}

ū1^b

ū1^b = [illegible]

Λ[1,ν]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, −e7/4}	Λ[1,iν]{*, −i e7/4}	Λ[1,ν*]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, −e7/4}	Λ[1,ν†]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/4}
Λ[2,ν]{*, 0}	Λ[2,iν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{{e2, ν1, d2^r, ē2, d̄2^r}, e6/4}	Λ[3,iν]{*, i e6/4}	Λ[3,ν*]{{e2, ν1, d2^r, ē2, d̄2^r}, e6/4}	Λ[3,ν†]{{e2, ν1, d2^r, ē2, d̄2^r}, −e6/4}
Λ[4,ν]{*, 0}	Λ[4,iν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{{ū1^r, ū2^r}, e4/4 − i e5/4}	Λ[5,iν]{{ē3, d̄2^b}, e5/4 + i e4/4}	Λ[5,ν*]{{ū1^r, ū2^r}, e4/4 + i e5/4}	Λ[5,ν†]{{ū1^r, ū2^r}, −e4/4 + i e5/4}
Λ[6,ν]{*, 0}	Λ[6,iν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,iν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{{e2, ν1, d2^r, ē2, d̄2^r}, e6/4}	Λ[8,iν]{*, i e6/4}	Λ[8,ν*]{{e2, ν1, d2^r, ē2, d̄2^r}, e6/4}	Λ[8,ν†]{{e2, ν1, d2^r, ē2, d̄2^r}, −e6/4}
g[9,ν]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, −e7/288}	g[9,iν]{*, −i e7/288}	g[9,ν*]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, −e7/288}	g[9,ν†]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/288}
g[10,ν]{*, −i e7/288}	g[10,iν]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/288}	g[10,ν*]{*, i e7/288}	g[10,ν†]{*, i e7/288}
g[11,ν]{{ē3, d̄2^b}, e5/288 − i e4/288}	g[11,iν]{{ū1^r, ū2^r}, e4/288 + i e5/288}	g[11,ν*]{{ē3, d̄2^b}, e5/288 + i e4/288}	g[11,ν†]{{d̄2^b, ē3}, −e5/288 + i e4/288}
g[12,ν]{{ū1^r, ū2^r}, −e4/288 − i e5/288}	g[12,iν]{{ē3, d̄2^b}, e5/288 − i e4/288}	g[12,ν*]{{ū1^r, ū2^r}, −e4/288 + i e5/288}	g[12,ν†]{{ū1^r, ū2^r}, e4/288 + i e5/288}
g[13,ν]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, −e3/48 − i e1/48}	g[13,iν]{*, e1/48 − i e3/48}	g[13,ν*]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, −e3/48 + i e1/48}	g[13,ν†]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/48 + i e1/48}
g[14,ν]{*, e1/288 + i e3/288}	g[14,iν]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, −e3/288 + i e1/288}	g[14,ν*]{*, −e1/288 − i e3/288}	g[14,ν†]{*, −e1/288 − i e3/288}

ē2

$\overline{e_2} = 2 e_6 - 2 i e_2$

Λ[1,ν]{*, 0}	Λ[1,iν]{*, 0}	Λ[1,ν*]{*, −2 i e7}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,iν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, e2 + i e6}	Λ[3,iν]{{ē2, d̄2^r}, −e6 + i e2}	Λ[3,ν*]{*, e2 + i e6}	Λ[3,ν†]{*, −e2 − i e6}
Λ[4,ν]{*, 0}	Λ[4,iν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{{ē3, d̄2^b}, 2e5 + 2 i e4}	Λ[5,iν]{*, −2e4 + 2 i e5}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{{d̄2^b, ē3}, −2e5 − 2 i e4}
Λ[6,ν]{*, 0}	Λ[6,iν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,iν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, e2 + i e6}	Λ[8,iν]{{ē2, d̄2^r}, −e6 + i e2}	Λ[8,ν*]{*, e2 + i e6}	Λ[8,ν†]{*, −e2 − i e6}
g[9,ν]{*, −i e7/36}	g[9,iν]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/36}	g[9,ν*]{*, 0}	g[9,ν†]{*, i e7/36}
g[10,ν]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/36}	g[10,iν]{*, i e7/36}	g[10,ν*]{*, 0}	g[10,ν†]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, −e7/36}
g[11,ν]{*, 0}	g[11,iν]{*, 0}	g[11,ν*]{{u1^r, u2^r}, −e4/36 + i e5/36}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,iν]{*, 0}	g[12,ν*]{{d2^b, e3}, −e5/36 − i e4/36}	g[12,ν†]{*, 0}
g[13,ν]{*, e1/6 − i e3/6}	g[13,iν]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/6 + i e1/6}	g[13,ν*]{*, 0}	g[13,ν†]{*, −e1/6 + i e3/6}
g[14,ν]{*, 0}	g[14,iν]{*, 0}	g[14,ν*]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/36 + i e1/36}	g[14,ν†]{*, 0}

ν̄2

$\overline{\nu_2} = 3 e_7$

Λ[1,ν]{*, −3e2/2 − 3 i e6/2}	Λ[1,iν]{*, 3e6/2 − 3 i e2/2}	Λ[1,ν*]{*, −3e2/2 + 3 i e6/2}	Λ[1,ν†]{*, 3e2/2 + 3 i e6/2}
Λ[2,ν]{*, 0}	Λ[2,iν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,iν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,iν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 3e3/2 − 3 i e1/2}	Λ[5,iν]{*, 3e1/2 + 3 i e3/2}	Λ[5,ν*]{*, 3e3/2 + 3 i e1/2}	Λ[5,ν†]{*, −3e3/2 + 3 i e1/2}
Λ[6,ν]{*, 0}	Λ[6,iν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,iν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,iν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, −e2/48 + i e6/48}	g[9,iν]{{e2, d2^r}, −e6/48 − i e2/48}	g[9,ν*]{*, −e2/48 − i e6/48}	g[9,ν†]{*, e2/48 − i e6/48}
g[10,ν]{{e2, d2^r}, −e6/48 − i e2/48}	g[10,iν]{*, e2/48 − i e6/48}	g[10,ν*]{{ē2, d̄2^r}, −e6/48 + i e2/48}	g[10,ν†]{{e2, d2^r}, e6/48 + i e2/48}
g[11,ν]{*, e1/48 − i e3/48}	g[11,iν]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/48 + i e1/48}	g[11,ν*]{*, e1/48 + i e3/48}	g[11,ν†]{*, −e1/48 + i e3/48}
g[12,ν]{{d1^r, u3^g, d3^b, ū1^g, ū2^g}, −e3/48 − i e1/48}	g[12,iν]{*, e1/48 − i e3/48}	g[12,ν*]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3/48 + i e1/48}	g[12,ν†]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/48 + i e1/48}
g[13,ν]{{ū1^r, ū2^r}, e4/8 + i e5/8}	g[13,iν]{{d̄2^b, ē3}, −e5/8 + i e4/8}	g[13,ν*]{{u1^r, u2^r}, e4/8 − i e5/8}	g[13,ν†]{{ū1^r, ū2^r}, −e4/8 − i e5/8}
g[14,ν]{{d2^b, e3}, −e5/48 − i e4/48}	g[14,iν]{{u1^r, u2^r}, e4/48 − i e5/48}	g[14,ν*]{{d̄2^b, ē3}, −e5/48 + i e4/48}	g[14,ν†]{{e3, d2^b}, e5/48 + i e4/48}

d̄2^r

$\overline{d_2^r} = 2 e_6 - 2 i e_2$

Λ[1,ν]{*, 0}	Λ[1,iν]{*, 0}	Λ[1,ν*]{*, −2 i e7}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,iν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, e2 + i e6}	Λ[3,iν]{{ē2, d̄2^r}, −e6 + i e2}	Λ[3,ν*]{*, e2 + i e6}	Λ[3,ν†]{*, −e2 − i e6}
Λ[4,ν]{*, 0}	Λ[4,iν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{{ē3, d̄2^b}, 2e5 + 2 i e4}	Λ[5,iν]{*, −2e4 + 2 i e5}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{{d̄2^b, ē3}, −2e5 − 2 i e4}
Λ[6,ν]{*, 0}	Λ[6,iν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,iν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, e2 + i e6}	Λ[8,iν]{{ē2, d̄2^r}, −e6 + i e2}	Λ[8,ν*]{*, e2 + i e6}	Λ[8,ν†]{*, −e2 − i e6}
g[9,ν]{*, −i e7/36}	g[9,iν]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/36}	g[9,ν*]{*, 0}	g[9,ν†]{*, i e7/36}
g[10,ν]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/36}	g[10,iν]{*, i e7/36}	g[10,ν*]{*, 0}	g[10,ν†]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, −e7/36}
g[11,ν]{*, 0}	g[11,iν]{*, 0}	g[11,ν*]{{u1^r, u2^r}, −e4/36 + i e5/36}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,iν]{*, 0}	g[12,ν*]{{d2^b, e3}, −e5/36 − i e4/36}	g[12,ν†]{*, 0}
g[13,ν]{*, e1/6 − i e3/6}	g[13,iν]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/6 + i e1/6}	g[13,ν*]{*, 0}	g[13,ν†]{*, −e1/6 + i e3/6}
g[14,ν]{*, 0}	g[14,iν]{*, 0}	g[14,ν*]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/36 + i e1/36}	g[14,ν†]{*, 0}

d̄2^g

$\overline{d_2^g} = 2 e_7 - 2 i$

Λ[1,ν]{*, −e2 − i e6}	Λ[1,iν]{{ē2, d̄2^r}, e6 − i e2}	Λ[1,ν*]{*, −e2 + i e6}	Λ[1,ν†]{*, e2 + i e6}
Λ[2,ν]{*, 0}	Λ[2,iν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,iν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,iν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{{u1^g, u2^g, d̄3^b, d̄1^r, ū3^g}, e3 − i e1}	Λ[5,iν]{*, e1 + i e3}	Λ[5,ν*]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3 + i e1}	Λ[5,ν†]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3 + i e1}
Λ[6,ν]{*, 0}	Λ[6,iν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,iν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,iν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, −e2/72 + i e6/72}	g[9,iν]{{e2, d2^r}, −e6/72 − i e2/72}	g[9,ν*]{*, −e2/72 − i e6/72}	g[9,ν†]{*, e2/72 − i e6/72}
g[10,ν]{{e2, d2^r}, −e6/72 − i e2/72}	g[10,iν]{*, e2/72 − i e6/72}	g[10,ν*]{{ē2, d̄2^r}, −e6/72 + i e2/72}	g[10,ν†]{{e2, d2^r}, e6/72 + i e2/72}
g[11,ν]{*, e1/72 − i e3/72}	g[11,iν]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/72 + i e1/72}	g[11,ν*]{*, e1/72 + i e3/72}	g[11,ν†]{*, −e1/72 + i e3/72}
g[12,ν]{{d1^r, u3^g, d3^b, ū1^g, ū2^g}, −e3/72 − i e1/72}	g[12,iν]{*, e1/72 − i e3/72}	g[12,ν*]{{d̄1^r, ū3^g, u1^g, u2^g, d̄3^b}, −e3/72 + i e1/72}	g[12,ν†]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/72 + i e1/72}
g[13,ν]{{ū1^r, ū2^r}, e4/12 + i e5/12}	g[13,iν]{{d̄2^b, ē3}, −e5/12 + i e4/12}	g[13,ν*]{{u1^r, u2^r}, e4/12 − i e5/12}	g[13,ν†]{{ū1^r, ū2^r}, −e4/12 − i e5/12}
g[14,ν]{{d2^b, e3}, −e5/72 − i e4/72}	g[14,iν]{{u1^r, u2^r}, e4/72 − i e5/72}	g[14,ν*]{{d̄2^b, ē3}, −e5/72 + i e4/72}	g[14,ν†]{{e3, d2^b}, e5/72 + i e4/72}

d̄2^b

$\overline{d_2^b} = -2 e_5 + 2 i e_4$

Λ[1,ν]{*, −2e1 + 2 i e3}	Λ[1,iν]{*, −2e3 − 2 i e1}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, 2e1 − 2 i e3}
Λ[2,ν]{*, 0}	Λ[2,iν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{{ū1^r, ū2^r}, −e4 − i e5}	Λ[3,iν]{{ē3, d̄2^b}, e5 − i e4}	Λ[3,ν*]{{ū1^r, ū2^r}, −e4 − i e5}	Λ[3,ν†]{{ū1^r, ū2^r}, e4 + i e5}
Λ[4,ν]{*, 0}	Λ[4,iν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{{e2, d2^r}, 2e6 + 2 i e2}	Λ[5,iν]{*, −2e2 + 2 i e6}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{{e2, d2^r}, −2e6 − 2 i e2}
Λ[6,ν]{*, 0}	Λ[6,iν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,iν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{{ū1^r, ū2^r}, −e4 − i e5}	Λ[8,iν]{{ē3, d̄2^b}, e5 − i e4}	Λ[8,ν*]{{ū1^r, ū2^r}, −e4 − i e5}	Λ[8,ν†]{{ū1^r, ū2^r}, e4 + i e5}
g[9,ν]{*, 0}	g[9,iν]{*, 0}	g[9,ν*]{*, −e1/36 + i e3/36}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,iν]{*, 0}	g[10,ν*]{{d3^b, ū1^g, ū2^g, d1^r, u3^g}, e3/36 + i e1/36}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,iν]{*, 0}	g[11,ν*]{*, −e2/36 + i e6/36}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,iν]{*, 0}	g[12,ν*]{{e2, d2^r}, −e6/36 − i e2/36}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,iν]{*, 0}	g[13,ν*]{*, −i e7/6}	g[13,ν†]{*, 0}
g[14,ν]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, −e7/36}	g[14,iν]{*, −i e7/36}	g[14,ν*]{*, 0}	g[14,ν†]{{ν2, ν1, d2^g, ν̄2, d̄2^g}, e7/36}

$\overline{u^{r}_{2}}$

$\overline{u^{r}_{2}} = 2\left(-\frac{e4}{4} - \frac{i\,e5}{4}\right) - 2\left(\frac{e4}{4} + \frac{i\,e5}{4}\right)$

Λ[1,ν]{{$\overline{d^{r}_{1}}$, $\overline{u^{g}_{3}}$, $\overline{d^{b}_{3}}$, $\overline{u^{g}_{1}}$, $\overline{u^{g}_{2}}$}, −e3 − i e1}	Λ[1,i ν]{*, e1 − i e3}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{{$\overline{d^{b}_{3}}$, $\overline{u^{g}_{1}}$, $\overline{u^{g}_{2}}$, $\overline{d^{r}_{1}}$, $\overline{u^{g}_{3}}$}, e3 + i e1}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{{e3, $\overline{d^{b}_{2}}$}, $\frac{e5}{2} - \frac{i e4}{2}$}	Λ[3,i ν]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, $\frac{e4}{2} + \frac{i e5}{2}$}	Λ[3,ν*]{{e3, $\overline{d^{b}_{2}}$}, $\frac{e5}{2} - \frac{i e4}{2}$}	Λ[3,ν†]{{$\overline{d^{b}_{2}}$, e3}, $-\frac{e5}{2} + \frac{i e4}{2}$}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, −e2 + i e6}	Λ[5,i ν]{{e2, $\overline{d^{r}_{2}}$}, −e6 − i e2}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{*, e2 − i e6}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{{e3, $\overline{d^{b}_{2}}$}, $\frac{e5}{2} - \frac{i e4}{2}$}	Λ[8,i ν]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, $\frac{e4}{2} + \frac{i e5}{2}$}	Λ[8,ν*]{{e3, $\overline{d^{b}_{2}}$}, $\frac{e5}{2} - \frac{i e4}{2}$}	Λ[8,ν†]{{$\overline{d^{b}_{2}}$, e3}, $-\frac{e5}{2} + \frac{i e4}{2}$}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{{$\overline{d^{r}_{1}}$, $\overline{u^{g}_{3}}$, $\overline{d^{b}_{3}}$, $\overline{u^{g}_{1}}$, $\overline{u^{g}_{2}}$}, $-\frac{e3}{72} - \frac{i e1}{72}$}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{*, $-\frac{e1}{72} + \frac{i e3}{72}$}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,i ν]{*, 0}	g[11,ν*]{{e2, $\overline{d^{r}_{2}}$}, $-\frac{e6}{72} - \frac{i e2}{72}$}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,i ν]{*, 0}	g[12,ν*]{*, $\frac{e2}{72} - \frac{i e6}{72}$}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $\frac{e7}{12}$}	g[13,ν†]{*, 0}
g[14,ν]{*, $-\frac{i e7}{72}$}	g[14,i ν]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $\frac{e7}{72}$}	g[14,ν*]{*, 0}	g[14,ν†]{*, $\frac{i e7}{72}$}

$\overline{u^{g}_{2}}$

$\overline{u^{g}_{2}} = 4\left(\frac{e3}{4} + \frac{i\,e1}{4}\right)$

Λ[1,ν]{*, 0}	Λ[1,i ν]{*, 0}	Λ[1,ν*]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, −e4 − i e5}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, $\frac{e1}{2} - \frac{i e3}{2}$}	Λ[3,i ν]{{d^{b}_{3}, $\overline{u^{g}_{1}}$, $\overline{u^{g}_{2}}$, d^{r}_{1}, u^{g}_{3}}, $\frac{e3}{2} + \frac{i e1}{2}$}	Λ[3,ν*]{*, $\frac{e1}{2} - \frac{i e3}{2}$}	Λ[3,ν†]{*, $-\frac{e1}{2} + \frac{i e3}{2}$}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, −e7}	Λ[5,i ν]{*, −i e7}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, e7}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, $\frac{e1}{2} - \frac{i e3}{2}$}	Λ[8,i ν]{{d^{b}_{3}, $\overline{u^{g}_{1}}$, $\overline{u^{g}_{2}}$, d^{r}_{1}, u^{g}_{3}}, $\frac{e3}{2} + \frac{i e1}{2}$}	Λ[8,ν*]{*, $\frac{e1}{2} - \frac{i e3}{2}$}	Λ[8,ν†]{*, $-\frac{e1}{2} + \frac{i e3}{2}$}
g[9,ν]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, $-\frac{e4}{72} - \frac{i e5}{72}$}	g[9,i ν]{{e3, $\overline{d^{b}_{2}}$}, $\frac{e5}{72} - \frac{i e4}{72}$}	g[9,ν*]{*, 0}	g[9,ν†]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, $\frac{e4}{72} + \frac{i e5}{72}$}
g[10,ν]{{e3, $\overline{d^{b}_{2}}$}, $\frac{e5}{72} - \frac{i e4}{72}$}	g[10,i ν]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, $\frac{e4}{72} + \frac{i e5}{72}$}	g[10,ν*]{*, 0}	g[10,ν†]{{$\overline{d^{b}_{2}}$, e3}, $-\frac{e5}{72} + \frac{i e4}{72}$}
g[11,ν]{*, 0}	g[11,i ν]{*, 0}	g[11,ν*]{*, $-\frac{i e7}{72}$}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,i ν]{*, 0}	g[12,ν*]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $\frac{e7}{72}$}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, $-\frac{e2}{12} - \frac{i e6}{12}$}	g[13,ν†]{*, 0}
g[14,ν]{{$\overline{e_2}$, $\overline{d^{r}_{2}}$}, $-\frac{e6}{72} + \frac{i e2}{72}$}	g[14,i ν]{*, $-\frac{e2}{72} - \frac{i e6}{72}$}	g[14,ν*]{*, 0}	g[14,ν†]{{$\overline{e_2}$, $\overline{d^{r}_{2}}$}, $\frac{e6}{72} - \frac{i e2}{72}$}

$\overline{u^{b}_{2}}$

$\overline{u^{b}_{2}} = -\frac{e2}{2}$

Λ[1,ν]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $-\frac{e7}{4}$}	Λ[1,i ν]{*, $-\frac{i e7}{4}$}	Λ[1,ν*]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $-\frac{e7}{4}$}	Λ[1,ν†]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $\frac{e7}{4}$}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{{e2, ν_1, d^{r}_{2}, $\overline{e_2}$, $\overline{d^{r}_{2}}$}, $\frac{e6}{4}$}	Λ[3,i ν]{*, $\frac{i e6}{4}$}	Λ[3,ν*]{{e2, ν_1, d^{r}_{2}, $\overline{e_2}$, $\overline{d^{r}_{2}}$}, $\frac{e6}{4}$}	Λ[3,ν†]{{e2, ν_1, d^{r}_{2}, $\overline{e_2}$, $\overline{d^{r}_{2}}$}, $-\frac{e6}{4}$}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, $\frac{e4}{4} - \frac{i e5}{4}$}	Λ[5,i ν]{{e3, $\overline{d^{b}_{2}}$}, $\frac{e5}{4} + \frac{i e4}{4}$}	Λ[5,ν*]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, $\frac{e4}{4} + \frac{i e5}{4}$}	Λ[5,ν†]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, $-\frac{e4}{4} + \frac{i e5}{4}$}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{{e2, ν_1, d^{r}_{2}, $\overline{e_2}$, $\overline{d^{r}_{2}}$}, $\frac{e6}{4}$}	Λ[8,i ν]{*, $\frac{i e6}{4}$}	Λ[8,ν*]{{e2, ν_1, d^{r}_{2}, $\overline{e_2}$, $\overline{d^{r}_{2}}$}, $\frac{e6}{4}$}	Λ[8,ν†]{{e2, ν_1, d^{r}_{2}, $\overline{e_2}$, $\overline{d^{r}_{2}}$}, $-\frac{e6}{4}$}
g[9,ν]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $-\frac{e7}{288}$}	g[9,i ν]{*, $-\frac{i e7}{288}$}	g[9,ν*]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $-\frac{e7}{288}$}	g[9,ν†]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $\frac{e7}{288}$}
g[10,ν]{*, $-\frac{i e7}{288}$}	g[10,i ν]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $\frac{e7}{288}$}	g[10,ν*]{*, $\frac{i e7}{288}$}	g[10,ν†]{*, $\frac{i e7}{288}$}
g[11,ν]{{e3, $\overline{d^{b}_{2}}$}, $\frac{e5}{288} - \frac{i e4}{288}$}	g[11,i ν]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, $\frac{e4}{288} + \frac{i e5}{288}$}	g[11,ν*]{{e3, d^{b}_{2}}, $\frac{e5}{288} + \frac{i e4}{288}$}	g[11,ν†]{{$\overline{d^{b}_{2}}$, e3}, $-\frac{e5}{288} + \frac{i e4}{288}$}
g[12,ν]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, $-\frac{e4}{288} - \frac{i e5}{288}$}	g[12,i ν]{{e3, $\overline{d^{b}_{2}}$}, $\frac{e5}{288} - \frac{i e4}{288}$}	g[12,ν*]{{u^{r}_{1}, u^{r}_{2}}, $-\frac{e4}{288} + \frac{i e5}{288}$}	g[12,ν†]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, $\frac{e4}{288} + \frac{i e5}{288}$}
g[13,ν]{{$\overline{d^{r}_{1}}$, $\overline{u^{g}_{3}}$, d^{b}_{3}, $\overline{u^{g}_{1}}$, $\overline{u^{g}_{2}}$}, $-\frac{e3}{48} - \frac{i e1}{48}$}	g[13,i ν]{*, $\frac{e1}{48} - \frac{i e3}{48}$}	g[13,ν*]{{$\overline{d^{r}_{1}}$, $\overline{u^{g}_{3}}$, u^{g}_{1}, u^{g}_{2}, $\overline{d^{b}_{3}}$}, $-\frac{e3}{48} + \frac{i e1}{48}$}	g[13,ν†]{{d^{b}_{3}, $\overline{u^{g}_{1}}$, $\overline{u^{g}_{2}}$, d^{r}_{1}, u^{g}_{3}}, $\frac{e3}{48} + \frac{i e1}{48}$}
g[14,ν]{*, $\frac{e1}{288} + \frac{i e3}{288}$}	g[14,i ν]{{$\overline{d^{r}_{1}}$, $\overline{u^{g}_{3}}$, u^{g}_{1}, u^{g}_{2}, $\overline{d^{b}_{3}}$}, $-\frac{e3}{288} + \frac{i e1}{288}$}	g[14,ν*]{*, $\frac{e1}{288} - \frac{i e3}{288}$}	g[14,ν†]{*, $-\frac{e1}{288} - \frac{i e3}{288}$}

$\overline{e_3}$

$\overline{e_3} = 2e5 - 2\,i\,e4$

Λ[1,ν]{*, 2e1 − 2 i e3}	Λ[1,i ν]{*, 2e3 + 2 i e1}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, −2e1 + 2 i e3}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, e4 + i e5}	Λ[3,i ν]{{$\overline{d^{b}_{2}}$, e3}, −e5 + i e4}	Λ[3,ν*]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, e4 + i e5}	Λ[3,ν†]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, −e4 − i e5}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{{e2, $\overline{d^{r}_{2}}$}, −2e6 − 2 i e2}	Λ[5,i ν]{*, 2e2 − 2 i e6}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{{e2, $\overline{d^{r}_{2}}$}, 2e6 + 2 i e2}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, e4 + i e5}	Λ[8,i ν]{{$\overline{d^{b}_{2}}$, e3}, −e5 + i e4}	Λ[8,ν*]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, e4 + i e5}	Λ[8,ν†]{{$\overline{u^{r}_{1}}$, $\overline{u^{r}_{2}}$}, −e4 − i e5}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{*, $\frac{e1}{36} - \frac{i e3}{36}$}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{{$\overline{d^{r}_{1}}$, $\overline{u^{g}_{3}}$, $\overline{d^{b}_{3}}$, $\overline{u^{g}_{1}}$, $\overline{u^{g}_{2}}$}, $-\frac{e3}{36} - \frac{i e1}{36}$}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,i ν]{*, 0}	g[11,ν*]{*, $\frac{e2}{36} - \frac{i e6}{36}$}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,i ν]{*, 0}	g[12,ν*]{{e2, $\overline{d^{r}_{2}}$}, $\frac{e6}{36} + \frac{i e2}{36}$}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, $\frac{i e7}{6}$}	g[13,ν†]{*, 0}
g[14,ν]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $\frac{e7}{36}$}	g[14,i ν]{*, $\frac{i e7}{36}$}	g[14,ν*]{*, 0}	g[14,ν†]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $-\frac{e7}{36}$}

$\overline{\nu_3}$

$\overline{\nu_3} = -2\,i$

Λ[1,ν]{*, 0}	Λ[1,i ν]{*, 0}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,i ν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,i ν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{*, 0}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{*, 0}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,i ν]{*, 0}	g[11,ν*]{*, 0}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,i ν]{*, 0}	g[12,ν*]{*, 0}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, 0}	g[13,ν†]{*, 0}
g[14,ν]{*, 0}	g[14,i ν]{*, 0}	g[14,ν*]{*, 0}	g[14,ν†]{*, 0}

$\overline{d^{r}_{3}}$

$\overline{d^{r}_{3}} = 0$

Λ[1,ν]{*, 0}	Λ[1,i ν]{*, 0}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,i ν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,i ν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{*, 0}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{*, 0}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,i ν]{*, 0}	g[11,ν*]{*, 0}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,i ν]{*, 0}	g[12,ν*]{*, 0}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, 0}	g[13,ν†]{*, 0}
g[14,ν]{*, 0}	g[14,i ν]{*, 0}	g[14,ν*]{*, 0}	g[14,ν†]{*, 0}

$\overline{d^{g}_{3}}$

$\overline{d^{g}_{3}} = 0$

Λ[1,ν]{*, 0}	Λ[1,i ν]{*, 0}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,i ν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,i ν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{*, 0}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{*, 0}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,i ν]{*, 0}	g[11,ν*]{*, 0}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,i ν]{*, 0}	g[12,ν*]{*, 0}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, 0}	g[13,ν†]{*, 0}
g[14,ν]{*, 0}	g[14,i ν]{*, 0}	g[14,ν*]{*, 0}	g[14,ν†]{*, 0}

$\overline{d^{b}_{3}}$

$\overline{d^{b}_{3}} = e3 - i\,e1$

Λ[1,ν]{{$\overline{u^{r}_{1}}$, u^{r}_{2}}, −e4 + i e5}	Λ[1,i ν]{{$\overline{d^{b}_{2}}$, e3}, −e5 − i e4}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{{u^{r}_{1}, u^{r}_{2}}, e4 − i e5}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, $\frac{e1}{2} + \frac{i e3}{2}$}	Λ[3,i ν]{{$\overline{d^{r}_{1}}$, $\overline{u^{g}_{3}}$, u^{g}_{1}, u^{g}_{2}, $\overline{d^{b}_{3}}$}, $-\frac{e3}{2} + \frac{i e1}{2}$}	Λ[3,ν*]{*, $\frac{e1}{2} + \frac{i e3}{2}$}	Λ[3,ν†]{*, $-\frac{e1}{2} - \frac{i e3}{2}$}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, −e7}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, $\frac{e1}{2} + \frac{i e3}{2}$}	Λ[8,i ν]{{$\overline{d^{r}_{1}}$, $\overline{u^{g}_{3}}$, u^{g}_{1}, u^{g}_{2}, $\overline{d^{b}_{3}}$}, $-\frac{e3}{2} + \frac{i e1}{2}$}	Λ[8,ν*]{*, $\frac{e1}{2} + \frac{i e3}{2}$}	Λ[8,ν†]{*, $-\frac{e1}{2} - \frac{i e3}{2}$}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{{u^{r}_{1}, u^{r}_{2}}, $-\frac{e4}{72} + \frac{i e5}{72}$}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{{e3, d^{b}_{2}}, $\frac{e5}{72} + \frac{i e4}{72}$}	g[10,ν†]{*, 0}
g[11,ν]{*, $\frac{i e7}{72}$}	g[11,i ν]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $-\frac{e7}{72}$}	g[11,ν*]{*, 0}	g[11,ν†]{*, $-\frac{i e7}{72}$}
g[12,ν]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $\frac{e7}{72}$}	g[12,i ν]{*, $\frac{i e7}{72}$}	g[12,ν*]{*, 0}	g[12,ν†]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $-\frac{e7}{72}$}
g[13,ν]{*, $-\frac{e2}{12} + \frac{i e6}{12}$}	g[13,i ν]{{e2, d^{r}_{2}}, $-\frac{e6}{12} - \frac{i e2}{12}$}	g[13,ν*]{*, 0}	g[13,ν†]{*, $\frac{e2}{12} - \frac{i e6}{12}$}
g[14,ν]{*, 0}	g[14,i ν]{*, 0}	g[14,ν*]{{e2, d^{r}_{2}}, $-\frac{e6}{72} - \frac{i e2}{72}$}	g[14,ν†]{*, 0}

$\overline{u^{r}_{3}}$

$\overline{u^{r}_{3}} = 0$

Λ[1,ν]{*, 0}	Λ[1,i ν]{*, 0}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,i ν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,i ν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{*, 0}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{*, 0}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,i ν]{*, 0}	g[11,ν*]{*, 0}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,i ν]{*, 0}	g[12,ν*]{*, 0}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, 0}	g[13,ν†]{*, 0}
g[14,ν]{*, 0}	g[14,i ν]{*, 0}	g[14,ν*]{*, 0}	g[14,ν†]{*, 0}

$\overline{u^{g}_{3}}$

$\overline{u^{g}_{3}} = -e3 + i\,e1$

Λ[1,ν]{{u^{r}_{1}, u^{r}_{2}}, e4 − i e5}	Λ[1,i ν]{{e3, d^{b}_{2}}, e5 + i e4}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{{u^{r}_{1}, u^{r}_{2}}, −e4 + i e5}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, $-\frac{e1}{2} - \frac{i e3}{2}$}	Λ[3,i ν]{{u^{g}_{1}, u^{g}_{2}, $\overline{d^{b}_{3}}$, $\overline{d^{r}_{1}}$, $\overline{u^{g}_{3}}$}, $\frac{e3}{2} - \frac{i e1}{2}$}	Λ[3,ν*]{*, $-\frac{e1}{2} - \frac{i e3}{2}$}	Λ[3,ν†]{*, $\frac{e1}{2} + \frac{i e3}{2}$}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, e7}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, $-\frac{e1}{2} - \frac{i e3}{2}$}	Λ[8,i ν]{{u^{g}_{1}, u^{g}_{2}, $\overline{d^{b}_{3}}$, $\overline{d^{r}_{1}}$, $\overline{u^{g}_{3}}$}, $\frac{e3}{2} - \frac{i e1}{2}$}	Λ[8,ν*]{*, $-\frac{e1}{2} - \frac{i e3}{2}$}	Λ[8,ν†]{*, $\frac{e1}{2} + \frac{i e3}{2}$}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{{u^{r}_{1}, u^{r}_{2}}, $\frac{e4}{72} - \frac{i e5}{72}$}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{{d^{b}_{2}, e3}, $-\frac{e5}{72} - \frac{i e4}{72}$}	g[10,ν†]{*, 0}
g[11,ν]{*, $-\frac{i e7}{72}$}	g[11,i ν]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $\frac{e7}{72}$}	g[11,ν*]{*, 0}	g[11,ν†]{*, $\frac{i e7}{72}$}
g[12,ν]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $-\frac{e7}{72}$}	g[12,i ν]{*, $-\frac{i e7}{72}$}	g[12,ν*]{*, 0}	g[12,ν†]{{ν_2, ν_1, d^{g}_{2}, $\overline{\nu_2}$, $\overline{d^{g}_{2}}$}, $\frac{e7}{72}$}
g[13,ν]{*, $\frac{e2}{12} - \frac{i e6}{12}$}	g[13,i ν]{{e2, d^{r}_{2}}, $\frac{e6}{12} + \frac{i e2}{12}$}	g[13,ν*]{*, 0}	g[13,ν†]{*, $-\frac{e2}{12} + \frac{i e6}{12}$}
g[14,ν]{*, 0}	g[14,i ν]{*, 0}	g[14,ν*]{{e2, d^{r}_{2}}, $\frac{e6}{72} + \frac{i e2}{72}$}	g[14,ν†]{*, 0}

$\overline{u^{b}_{3}}$

$\overline{u^{b}_{3}} = 0$

Λ[1,ν]{*, 0}	Λ[1,i ν]{*, 0}	Λ[1,ν*]{*, 0}	Λ[1,ν†]{*, 0}
Λ[2,ν]{*, 0}	Λ[2,i ν]{*, 0}	Λ[2,ν*]{*, 0}	Λ[2,ν†]{*, 0}
Λ[3,ν]{*, 0}	Λ[3,i ν]{*, 0}	Λ[3,ν*]{*, 0}	Λ[3,ν†]{*, 0}
Λ[4,ν]{*, 0}	Λ[4,i ν]{*, 0}	Λ[4,ν*]{*, 0}	Λ[4,ν†]{*, 0}
Λ[5,ν]{*, 0}	Λ[5,i ν]{*, 0}	Λ[5,ν*]{*, 0}	Λ[5,ν†]{*, 0}
Λ[6,ν]{*, 0}	Λ[6,i ν]{*, 0}	Λ[6,ν*]{*, 0}	Λ[6,ν†]{*, 0}
Λ[7,ν]{*, 0}	Λ[7,i ν]{*, 0}	Λ[7,ν*]{*, 0}	Λ[7,ν†]{*, 0}
Λ[8,ν]{*, 0}	Λ[8,i ν]{*, 0}	Λ[8,ν*]{*, 0}	Λ[8,ν†]{*, 0}
g[9,ν]{*, 0}	g[9,i ν]{*, 0}	g[9,ν*]{*, 0}	g[9,ν†]{*, 0}
g[10,ν]{*, 0}	g[10,i ν]{*, 0}	g[10,ν*]{*, 0}	g[10,ν†]{*, 0}
g[11,ν]{*, 0}	g[11,i ν]{*, 0}	g[11,ν*]{*, 0}	g[11,ν†]{*, 0}
g[12,ν]{*, 0}	g[12,i ν]{*, 0}	g[12,ν*]{*, 0}	g[12,ν†]{*, 0}
g[13,ν]{*, 0}	g[13,i ν]{*, 0}	g[13,ν*]{*, 0}	g[13,ν†]{*, 0}
g[14,ν]{*, 0}	g[14,i ν]{*, 0}	g[14,ν*]{*, 0}	g[14,ν†]{*, 0}

IX. Addendum : Towards a complete description

Working with Quaternions

In[2745]:=
```
γ1 = I e1; γ2 = I e2; γ3 = I e3; γ4 = I e4; γ5 = I e5; γ6 = I e6; γ7 = I e7; γ8 = J e7; γ9 = K e7;
```

In[2746]:=
```
ToQuaternion@γ# & /@ Range@9
```

Out[2746]= {Quaternion(0, e_1, 0, 0), Quaternion(0, e_2, 0, 0), Quaternion(0, e_3, 0, 0), Quaternion(0, e_4, 0, 0), Quaternion(0, e_5, 0, 0), Quaternion(0, e_6, 0, 0), Quaternion(0, e_7, 0, 0), Quaternion(0, 0, e_7, 0), Quaternion(0, 0, 0, e_7)}

In[2747]:=
```
fff@{γ1, γ2, γ3, γ4, γ5, γ6, γ7}
```

Out[2747]= i

In[2748]:=
```
ToQuaternion[fff@{γ1, γ2, γ3, γ4, γ5, γ6, γ7}]
ToQuaternion@γ8
%% ** %
```

Out[2748]= Quaternion(0, 1, 0, 0)

Out[2749]= Quaternion(0, 0, e_7, 0)

Out[2750]= Quaternion(0, 0, 0, e_7)

In[2751]:=
```
fff@{γ1, γ2, γ3, γ4, γ5, γ6, γ7} ** γ8
```

Out[2751]= $K e_7$

In[2752]:=
```
Table[γn ⊖ γm, {n, 8}, {m, 8}]
```

Out[2752]=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2\,i\,J \\ 0 & 0 & 0 & 0 & 0 & 0 & -2\,i\,J & -2\,J^2 \end{pmatrix}$$

The Setup for Multi – Actions

In[2753]:=
```
P1 = 1/2 (1 - γ9)
```

Out[2753]= $\frac{1}{2}(1 - K e_7)$

In[2754]:=
```
P2 = 1/2 (1 + γ9)
```

Out[2754]= $\frac{1}{2}(K e_7 + 1)$

In[2755]:=
```
m†[a_, b_] := Sum[(f@{Pi, a} ** f@{Pi, b} + f@{Pi, b} ** f@{Pi, a†}), {i, 1, 1}];

m̃[a_, b_] := Sum[(f@{Pi, a} ** f@{Pi, b} + f@{Pi, b} ** f@{Pi, ã}), {i, 1, 1}];
```

$$m^\dagger[a\_, b\_] := \sum_{i=1}^{1} \left(f@\{P_i, a\} ** f@\{P_i, b\} + f@\{P_i, b\} ** f@\{P_i, a^\dagger\} \right);$$

$$\tilde{m}[a\_, b\_] := \sum_{i=1}^{1} \left(f@\{P_i, a\} ** f@\{P_i, b\} + f@\{P_i, b\} ** f@\{P_i, \tilde{a}\} \right);$$

In[2757]:=

$$m^\dagger[a\_, b\_] := \sum_{i=1}^{1} \left(f@\{P_i, a, P_i, b\} + f@\{P_i, b, P_i, a^\dagger\} \right);$$

$$\tilde{m}[a\_, b\_] := \sum_{i=1}^{1} \left(f@\{P_i, a, P_i, b\} + f@\{P_i, b, P_i, \tilde{a}\} \right);$$

In[2759]:= `m†[ν, b]`

Out[2759]= $f\left(\left\{\frac{1}{2}(1-K e_7),\ b,\ \frac{1}{2}(1-K e_7),\ -\frac{1}{2}-\frac{i e_7}{2}\right\}\right)+f\left(\left\{\frac{1}{2}(1-K e_7),\ \frac{1}{2}(1+i e_7),\ \frac{1}{2}(1-K e_7),\ b\right\}\right)$

In[2760]:= `m†[ν, ω]`

Out[2760]= $f\left(\left\{\frac{1}{2}(1-K e_7),\ \frac{1}{2}(-e_7+i),\ \frac{1}{2}(1-K e_7),\ -\frac{1}{2}-\frac{i e_7}{2}\right\}\right)+f\left(\left\{\frac{1}{2}(1-K e_7),\ \frac{1}{2}(1+i e_7),\ \frac{1}{2}(1-K e_7),\ \frac{1}{2}(-e_7+i)\right\}\right)$

In[2761]:= `% /. f → fff`

Out[2761]= $\frac{1}{16}\big((-3+2 i) K^2 e_3+3 K^2 e_4-(3-2 i) K^2 e_5-(3-2 i) K^2 e_6-(2-i) K^2 e_7+(3+i) K^2+(3 K+(3-2 i)) K e_1+ (3 K+(3-2 i)) K e_2+3 K e_3+(3-2 i) K e_4+3 K e_5+3 K e_6+(3+3 i) K e_7+(4-i) K+2 e_7-2 i\big)+ \frac{1}{16}\big((3-2 i) K^2 e_3-3 K^2 e_4+(3-2 i) K^2 e_5+(3-2 i) K^2 e_6+(2-i) K^2 e_7-(3+i) K^2+(-3 K-(3-2 i)) K e_1+ (-3 K-(3-2 i)) K e_2-3 K e_3-(3-2 i) K e_4-3 K e_5-3 K e_6-(3+3 i) K e_7-(4-i) K-2 e_7+2 i\big)$

In[2762]:= `Simplify@%`

Out[2762]= 0

In[2763]:= `m̃[ν, ω]`

Out[2763]= $f\left(\left\{\frac{1}{2}(1-K e_7),\ \frac{1}{2}(-e_7+i),\ \frac{1}{2}(1-K e_7),\ \frac{1}{2}\widetilde{(1+i e_7)}\right\}\right)+f\left(\left\{\frac{1}{2}(1-K e_7),\ \frac{1}{2}(1+i e_7),\ \frac{1}{2}(1-K e_7),\ \frac{1}{2}(-e_7+i)\right\}\right)$