1) Set up variables for Koca 0906.2109v2 analysis

While this notebook will use octonions rather than quaternions, the quaternions will be shown to be seamlessly integrated into octonions. As long as we use only $\{e_0, e_1, e_2, e_3\}$ in our computations and select a multiplication table with the upper left quadrant being quaternions with the first triad={1,2,3} where $e_1 \circ e_2 = e_3$, the rest of the octonion elements $\{e_4, e_5, e_6, e_7\}$ are not referenced within the results.

We select one of 480 possible nontrivial octonion octonion multiplication tables . There are 48 possible with triad={1,2,3} where$e_1 \circ e_2 = e_3$. There are another 48 with the first triad={1,3,2} where $e_2 \circ e_1 = e_3$, this forms an alternate type of quaternion, which by convention is not generally used.

In[7519]:=

```
setFM[87, flip = True, split = 0];
triads
```

Out[7520]=

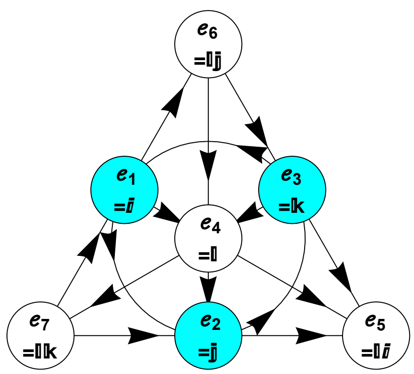
```
{{1, 2, 3}, {1, 4, 5}, {1, 6, 7}, {2, 6, 4}, {2, 5, 7}, {3, 4, 7}, {3, 5, 6}}
```

In[7521]:=

```
fanoPlane
fmDispN
```

Out[7521]=

Octonion Fano Plane



Out[7522]//MatrixForm=

0↦1	1	2	3	4	5	6	7
1	-1	3	-2	5	-4	7	-6
2	-3	-1	1	-6	7	4	-5
3	2	-1	-1	7	6	-5	-4
4	-5	6	-7	-1	1	-2	3
5	4	-7	-6	-1	-1	3	2
6	-7	-4	5	2	-3	-1	1
7	6	5	4	-3	-2	-1	-1

Define τ, σ, and p as used in the paper in terms of my base MTM symbol φ and show p in symbolic and numeric forms

In[7523]:=

```
σ = -φ;
τ = Φ = 1 / φ;
(**)
Clear[φ, Φ, p]
```

In[7526]:=

$$p = \frac{1}{2} (\tau + e_1 + \sigma e_3)$$

$$\% \;/.\; \varphi \to \left(\sqrt{5} - 1\right) / 2.$$

Out[7526]=

$$\frac{1}{2} \left(\frac{1}{\varphi} + e_1 - \varphi e_3\right)$$

Out[7527]=

$$\frac{1}{2} (1.61803 + e_1 - 0.618034\, e_3)$$

The octonion power results can be symbolically messy, so it is useful to use numeric values and rounding for comparison
.

We also can revert from numeric back to symbolic forms in some cases.

Function to $\tau \leftrightarrow \sigma$ in the 48 IcosO where $p^5 = \pm 1$.

In[7535]:=

```
(* This works for rounding with chop=10^-4
  after converting the input Quaternion or Octonion to a numeric list *)
switchστ@in_ := rndOct[oct2List@in] /.
    {-0.809 → x, 0.809 → -x, -0.309 → 0.809, 0.309 → -0.809} /. x → 0.309;
```

Show the numeric powers of $p^{n=0-5}$ & $\overline{p}^{n=0-5}$

Note : I will occasionally use a factor of 2 to simplify output readability

2) Demonstrate Quaternion & Octonion JGM-Built-In functions

Octonion Power octPower[ ] and Conjugation e_n^*

Note : we use an asterisk not the MTM conjugate symbol.

Need to define a few variables to play with....

Quaternion to Octonion conversion

Using the Conjugate symbol for Quaternions and the asterisk for octonions.

In[7557]:=

```
FromQuaternion@Λ1Quat*
```

Out[7557]=

(1.61803 − 1. i) − 1.61803 J − 1.61803 K

... and show the results in the two styles (IJKL vs. e_n)

In[7558]:=

```
IJKLstyle = True;
octStyle@quat2oct[Λ1Quat]*
```

Out[7559]=

(1.61803 − 1. i) − 1.61803 j − 1.61803 k

In[7560]:=

```
IJKLstyle = False;
octStyle@quat2oct[Λ1Quat]*
```

Out[7561]=

$1.61803 - 1.\, e_1 - 1.61803\, e_2 - 1.61803\, e_3$

Mixing and converting Quaternion to octonion math is included.

Note: I am using some of my predefined Split Real Even (SRE) E8 and D4 Lie groups here.

In[7562]:=
```
Quaternion[Sequence @@ pE8[d4[[2]]][[ ;; 4]]]
```
Out[7562]=
```
Quaternion[-1, 0, -1, 0]
```
In[7563]:=
```
Λ1Quat ** %
```
Out[7563]=
```
Quaternion[0., 0.618034, -3.23607, -2.61803]
```
In[7564]:=
```
quat2oct@%
```
Out[7564]=

$0. + 0.618034\, e_1 - 3.23607\, e_2 - 2.61803\, e_3$

Show that octonion math with an appropriately selected multiplication table (see above), matches the native Quaternion math.

Octonion functions include integer exponential powers, which simply iterate simple octonion product (octProduct[] or ∘)

In[7567]:=
```
octPower[quat2oct[Λ1Quat]*, #] & /@ Range@5 // MatrixForm
```
Out[7567]//MatrixForm=

$$\begin{pmatrix} 1.61803 - 1.\, e_1 - 1.61803\, e_2 - 1.61803\, e_3 \\ -3.61803 - 3.23607\, e_1 - 5.23607\, e_2 - 5.23607\, e_3 \\ -26.0344 - 1.61803\, e_1 - 2.61803\, e_2 - 2.61803\, e_3 \\ -52.2148 + 23.4164\, e_1 + 37.8885\, e_2 + 37.8885\, e_3 \\ 61.541 + 90.1033\, e_1 + 145.79\, e_2 + 145.79\, e_3 \end{pmatrix}$$

...with the same results as using native Quaternions.

In[7568]:=
```
Λ1Quat*# & /@ Range@5 // MatrixForm
FromQuaternion /@ % // MatrixForm
```
Out[7568]//MatrixForm=
```
Quaternion[1.61803, -1, -1.61803, -1.61803]
Quaternion[-3.61803, -3.23607, -5.23607, -5.23607]
Quaternion[-26.0344, -1.61803, -2.61803, -2.61803]
Quaternion[-52.2148, 23.4164, 37.8885, 37.8885]
Quaternion[61.541, 90.1033, 145.79, 145.79]
```
Out[7569]//MatrixForm=

$$\begin{pmatrix} (1.61803 - 1.\, i) - 1.61803\, J - 1.61803\, K \\ (-3.61803 - 3.23607\, i) - 5.23607\, J - 5.23607\, K \\ (-26.0344 - 1.61803\, i) - 2.61803\, J - 2.61803\, K \\ (-52.2148 + 23.4164\, i) + 37.8885\, J + 37.8885\, K \\ (61.541 + 90.1033\, i) + 145.79\, J + 145.79\, K \end{pmatrix}$$

3) Define some variables used in Koca papers.

Setup for an earlier 2007 paper :
Group theoretical analysis of 600 – cell and 120 – cell 4 D polytopes with quaternions ,

Simple Roots from 2007 paper after eq. 9

Setup for 2007 paper eq. 13

Setup for 2007 paper eq. 15

Setup for the 2009 paper analysis.

Define t1∘t2 with p being defined above in Section 1 to be used below in Section 7.

Weights

4) Define 120 vertex H4Φ Icosians (I in Table 1) using Quaternion(IcosQ) & Octonion(IcosO) forms.

This uses h4Φ from my E8->H4 folding matrix model.

In[7619]:=

```
IcosQ = Quaternion[Sequence @@ N[pC600[#][[ ;; 4]]]] & /@ h4Φ;
IcosO = quat2oct@# & /@ IcosQ;
{#, FullSimplify@FromQuaternion@IcosQ[[#]],
    IcosO[[#]]} & /@ Range@Length@IcosO // MatrixForm
```

Out[7621]//MatrixForm=

1	(-1. - 1. ⅈ) - 1. J - 1. K	-1. - 1. e_1 - 1. e_2 - 1. e_3
2	(-0.618034 - 1.61803 ⅈ) - 1. K	-0.618034 - 1.61803 e_1 - 1. e_3
3	-0.618034 - 1. J - 1.61803 K	-0.618034 - 1. e_2 - 1.61803 e_3
4	(-0.618034 - 1. ⅈ) - 1.61803 J	-0.618034 - 1. e_1 - 1.61803 e_2
5	(1. - 1. ⅈ) - 1. J - 1. K	1. - 1. e_1 - 1. e_2 - 1. e_3
6	(-1. - 0.618034 ⅈ) - 1.61803 K	-1. - 0.618034 e_1 - 1.61803 e_3
7	(-1. - 1.61803 ⅈ) - 0.618034 J	-1. - 1.61803 e_1 - 0.618034 e_2
8	(0.618034 - 1.61803 ⅈ) - 1. K	0.618034 - 1.61803 e_1 - 1. e_3
9	(-1. - 1. ⅈ) + 1. J - 1. K	-1. - 1. e_1 + 1. e_2 - 1. e_3
10	(-1. - 1.61803 ⅈ) + 0.618034 J	-1. - 1.61803 e_1 + 0.618034 e_2
11	-1. - 1.61803 J - 0.618034 K	-1. - 1.61803 e_2 - 0.618034 e_3
12	0.618034 - 1. J - 1.61803 K	0.618034 - 1. e_2 - 1.61803 e_3
13	(-1. + 0.618034 ⅈ) - 1.61803 K	-1. + 0.618034 e_1 - 1.61803 e_3
14	(-1. + 1. ⅈ) - 1. J - 1. K	-1. + 1. e_1 - 1. e_2 - 1. e_3
15	(0.618034 - 1. ⅈ) - 1.61803 J	0.618034 - 1. e_1 - 1.61803 e_2
16	-1. - 1.61803 J + 0.618034 K	-1. - 1.61803 e_2 + 0.618034 e_3
17	(-1. - 1. ⅈ) - 1. J + 1. K	-1. - 1. e_1 - 1. e_2 + 1. e_3
18	-2. + 0. ⅈ	-2.
19	(-1.61803 + 0.618034 ⅈ) - 1. J	-1.61803 + 0.618034 e_1 - 1. e_2
20	(0. + 0. ⅈ) - 2. J	0. - 2. e_2
21	(0. + 0.618034 ⅈ) - 1.61803 J - 1. K	0. + 0.618034 e_1 - 1.61803 e_2 - 1. e_3
22	(0. + 1.61803 ⅈ) - 1. J + 0.618034 K	0. + 1.61803 e_1 - 1. e_2 + 0.618034 e_3
23	(1.61803 + 0.618034 ⅈ) - 1. J	1.61803 + 0.618034 e_1 - 1. e_2
24	(0. + 0.618034 ⅈ) - 1.61803 J + 1. K	0. + 0.618034 e_1 - 1.61803 e_2 + 1. e_3
25	(0. + 1.61803 ⅈ) - 1. J - 0.618034 K	0. + 1.61803 e_1 - 1. e_2 - 0.618034 e_3
26	(-1.61803 - 1. ⅈ) + 0.618034 K	-1.61803 - 1. e_1 + 0.618034 e_3
27	(0. - 1.61803 ⅈ) - 1. J + 0.618034 K	0. - 1.61803 e_1 - 1. e_2 + 0.618034 e_3
28	0. - 2. ⅈ	0. - 2. e_1
29	(0. - 1. ⅈ) + 0.618034 J + 1.61803 K	0. - 1. e_1 + 0.618034 e_2 + 1.61803 e_3
30	(1.61803 - 1. ⅈ) + 0.618034 K	1.61803 - 1. e_1 + 0.618034 e_3
31	(0. - 1. ⅈ) - 0.618034 J + 1.61803 K	0. - 1. e_1 - 0.618034 e_2 + 1.61803 e_3
32	-1.61803 + 0.618034 J - 1. K	-1.61803 + 0.618034 e_2 - 1. e_3
33	(0. - 1. ⅈ) + 0.618034 J - 1.61803 K	0. - 1. e_1 + 0.618034 e_2 - 1.61803 e_3
34	(0. + 0. ⅈ) - 2. K	0. - 2. e_3
35	(0. + 0.618034 ⅈ) + 1.61803 J - 1. K	0. + 0.618034 e_1 + 1.61803 e_2 - 1. e_3
36	1.61803 + 0.618034 J - 1. K	1.61803 + 0.618034 e_2 - 1. e_3
37	(-1.61803 - 0.618034 ⅈ) - 1. J	-1.61803 - 0.618034 e_1 - 1. e_2
38	(-1.61803 - 1. ⅈ) - 0.618034 K	-1.61803 - 1. e_1 - 0.618034 e_3
39	-1.61803 - 0.618034 J - 1. K	-1.61803 - 0.618034 e_2 - 1. e_3
40	-1.61803 + 0.618034 J + 1. K	-1.61803 + 0.618034 e_2 + 1. e_3
41	(-1.61803 + 1. ⅈ) + 0.618034 K	-1.61803 + 1. e_1 + 0.618034 e_3
42	(-1.61803 + 0.618034 ⅈ) + 1. J	-1.61803 + 0.618034 e_1 + 1. e_2
43	(0. - 1.61803 ⅈ) - 1. J - 0.618034 K	0. - 1.61803 e_1 - 1. e_2 - 0.618034 e_3
44	(0. - 0.618034 ⅈ) - 1.61803 J - 1. K	0. - 0.618034 e_1 - 1.61803 e_2 - 1. e_3
45	(0. - 1. ⅈ) - 0.618034 J - 1.61803 K	0. - 1. e_1 - 0.618034 e_2 - 1.61803 e_3
46	(1. - 0.618034 ⅈ) - 1.61803 K	1. - 0.618034 e_1 - 1.61803 e_3

47	−0.618034 + 1. J − 1.61803 K	−0.618034 + 1. e_2 − 1.61803 e_3
48	(1. − 1.61803 ⅈ) − 0.618034 J	1. − 1.61803 e_1 − 0.618034 e_2
49	(−0.618034 − 1.61803 ⅈ) + 1. K	−0.618034 − 1.61803 e_1 + 1. e_3
50	(1. − 1. ⅈ) + 1. J − 1. K	1. − 1. e_1 + 1. e_2 − 1. e_3
51	(1. − 1.61803 ⅈ) + 0.618034 J	1. − 1.61803 e_1 + 0.618034 e_2
52	(−0.618034 − 1. ⅈ) + 1.61803 J	−0.618034 − 1. e_1 + 1.61803 e_2
53	1. − 1.61803 J − 0.618034 K	1. − 1.61803 e_2 − 0.618034 e_3
54	(−0.618034 + 1. ⅈ) − 1.61803 J	−0.618034 + 1. e_1 − 1.61803 e_2
55	(1. + 0.618034 ⅈ) − 1.61803 K	1. + 0.618034 e_1 − 1.61803 e_3
56	(1. + 1. ⅈ) − 1. J − 1. K	1. + 1. e_1 − 1. e_2 − 1. e_3
57	(−0.618034 + 1.61803 ⅈ) − 1. K	−0.618034 + 1.61803 e_1 − 1. e_3
58	1. − 1.61803 J + 0.618034 K	1. − 1.61803 e_2 + 0.618034 e_3
59	(1. − 1. ⅈ) − 1. J + 1. K	1. − 1. e_1 − 1. e_2 + 1. e_3
60	−0.618034 − 1. J + 1.61803 K	−0.618034 − 1. e_2 + 1.61803 e_3
61	0.618034 + 1. J − 1.61803 K	0.618034 + 1. e_2 − 1.61803 e_3
62	(−1. + 1. ⅈ) + 1. J − 1. K	−1. + 1. e_1 + 1. e_2 − 1. e_3
63	−1. + 1.61803 J − 0.618034 K	−1. + 1.61803 e_2 − 0.618034 e_3
64	(0.618034 − 1.61803 ⅈ) + 1. K	0.618034 − 1.61803 e_1 + 1. e_3
65	(−1. − 1. ⅈ) + 1. J + 1. K	−1. − 1. e_1 + 1. e_2 + 1. e_3
66	(−1. − 0.618034 ⅈ) + 1.61803 K	−1. − 0.618034 e_1 + 1.61803 e_3
67	(0.618034 − 1. ⅈ) + 1.61803 J	0.618034 − 1. e_1 + 1.61803 e_2
68	−1. + 1.61803 J + 0.618034 K	−1. + 1.61803 e_2 + 0.618034 e_3
69	(0.618034 + 1. ⅈ) − 1.61803 J	0.618034 + 1. e_1 − 1.61803 e_2
70	(−1. + 1.61803 ⅈ) − 0.618034 J	−1. + 1.61803 e_1 − 0.618034 e_2
71	(−1. + 1. ⅈ) − 1. J + 1. K	−1. + 1. e_1 − 1. e_2 + 1. e_3
72	(0.618034 + 1.61803 ⅈ) − 1. K	0.618034 + 1.61803 e_1 − 1. e_3
73	(−1. + 1.61803 ⅈ) + 0.618034 J	−1. + 1.61803 e_1 + 0.618034 e_2
74	0.618034 − 1. J + 1.61803 K	0.618034 − 1. e_2 + 1.61803 e_3
75	(−1. + 0.618034 ⅈ) + 1.61803 K	−1. + 0.618034 e_1 + 1.61803 e_3
76	(0. + 1. ⅈ) + 0.618034 J + 1.61803 K	0. + 1. e_1 + 0.618034 e_2 + 1.61803 e_3
77	(0. + 0.618034 ⅈ) + 1.61803 J + 1. K	0. + 0.618034 e_1 + 1.61803 e_2 + 1. e_3
78	(0. + 1.61803 ⅈ) + 1. J + 0.618034 K	0. + 1.61803 e_1 + 1. e_2 + 0.618034 e_3
79	(1.61803 − 0.618034 ⅈ) − 1. J	1.61803 − 0.618034 e_1 − 1. e_2
80	(1.61803 − 1. ⅈ) − 0.618034 K	1.61803 − 1. e_1 − 0.618034 e_3
81	1.61803 − 0.618034 J − 1. K	1.61803 − 0.618034 e_2 − 1. e_3
82	1.61803 + 0.618034 J + 1. K	1.61803 + 0.618034 e_2 + 1. e_3
83	(1.61803 + 1. ⅈ) + 0.618034 K	1.61803 + 1. e_1 + 0.618034 e_3
84	(1.61803 + 0.618034 ⅈ) + 1. J	1.61803 + 0.618034 e_1 + 1. e_2
85	−1.61803 − 0.618034 J + 1. K	−1.61803 − 0.618034 e_2 + 1. e_3
86	(0. − 0.618034 ⅈ) − 1.61803 J + 1. K	0. − 0.618034 e_1 − 1.61803 e_2 + 1. e_3
87	(0. + 0. ⅈ) + 2. K	0. + 2. e_3
88	(0. + 1. ⅈ) − 0.618034 J + 1.61803 K	0. + 1. e_1 − 0.618034 e_2 + 1.61803 e_3
89	1.61803 − 0.618034 J + 1. K	1.61803 − 0.618034 e_2 + 1. e_3
90	(0. + 1. ⅈ) + 0.618034 J − 1.61803 K	0. + 1. e_1 + 0.618034 e_2 − 1.61803 e_3
91	(−1.61803 + 1. ⅈ) − 0.618034 K	−1.61803 + 1. e_1 − 0.618034 e_3
92	(0. + 1. ⅈ) − 0.618034 J − 1.61803 K	0. + 1. e_1 − 0.618034 e_2 − 1.61803 e_3
93	0. + 2. ⅈ	0. + 2. e_1
94	(0. + 1.61803 ⅈ) + 1. J − 0.618034 K	0. + 1.61803 e_1 + 1. e_2 − 0.618034 e_3
95	(1.61803 + 1. ⅈ) − 0.618034 K	1.61803 + 1. e_1 − 0.618034 e_3
96	(0. − 1.61803 ⅈ) + 1. J + 0.618034 K	0. − 1.61803 e_1 + 1. e_2 + 0.618034 e_3
97	(0. − 0.618034 ⅈ) + 1.61803 J − 1. K	0. − 0.618034 e_1 + 1.61803 e_2 − 1. e_3
98	(−1.61803 − 0.618034 ⅈ) + 1. J	−1.61803 − 0.618034 e_1 + 1. e_2
99	(0. − 1.61803 ⅈ) + 1. J − 0.618034 K	0. − 1.61803 e_1 + 1. e_2 − 0.618034 e_3
100	(0. − 0.618034 ⅈ) + 1.61803 J + 1. K	0. − 0.618034 e_1 + 1.61803 e_2 + 1. e_3
101	(0. + 0. ⅈ) + 2. J	0. + 2. e_2
102	(1.61803 − 0.618034 ⅈ) + 1. J	1.61803 − 0.618034 e_1 + 1. e_2
103	2. + 0. ⅈ	2.
104	(1. + 1. ⅈ) + 1. J − 1. K	1. + 1. e_1 + 1. e_2 − 1. e_3
105	1. + 1.61803 J − 0.618034 K	1. + 1.61803 e_2 − 0.618034 e_3
106	[illegible]	[illegible]

106	$(-0.618034 + 1.\,i) + 1.61803\,J$	$-0.618034 + 1.\,e_1 + 1.61803\,e_2$
107	$(1. - 1.\,i) + 1.\,J + 1.\,K$	$1. - 1.\,e_1 + 1.\,e_2 + 1.\,e_3$
108	$(1. - 0.618034\,i) + 1.61803\,K$	$1. - 0.618034\,e_1 + 1.61803\,e_3$
109	$-0.618034 + 1.\,J + 1.61803\,K$	$-0.618034 + 1.\,e_2 + 1.61803\,e_3$
110	$1. + 1.61803\,J + 0.618034\,K$	$1. + 1.61803\,e_2 + 0.618034\,e_3$
111	$(1. + 1.61803\,i) - 0.618034\,J$	$1. + 1.61803\,e_1 - 0.618034\,e_2$
112	$(1. + 1.\,i) - 1.\,J + 1.\,K$	$1. + 1.\,e_1 - 1.\,e_2 + 1.\,e_3$
113	$(-0.618034 + 1.61803\,i) + 1.\,K$	$-0.618034 + 1.61803\,e_1 + 1.\,e_3$
114	$(1. + 1.61803\,i) + 0.618034\,J$	$1. + 1.61803\,e_1 + 0.618034\,e_2$
115	$(1. + 0.618034\,i) + 1.61803\,K$	$1. + 0.618034\,e_1 + 1.61803\,e_3$
116	$(-1. + 1.\,i) + 1.\,J + 1.\,K$	$-1. + 1.\,e_1 + 1.\,e_2 + 1.\,e_3$
117	$(0.618034 + 1.\,i) + 1.61803\,J$	$0.618034 + 1.\,e_1 + 1.61803\,e_2$
118	$0.618034 + 1.\,J + 1.61803\,K$	$0.618034 + 1.\,e_2 + 1.61803\,e_3$
119	$(0.618034 + 1.61803\,i) + 1.\,K$	$0.618034 + 1.61803\,e_1 + 1.\,e_3$
120	$(1. + 1.\,i) + 1.\,J + 1.\,K$	$1. + 1.\,e_1 + 1.\,e_2 + 1.\,e_3$

In[7622]:=

```
rndIcosO = rndOct[FullSimplify[# / 2]] & /@ IcosO;
```

5) Define the Unit Quaternions using my Octonion functions (eq. 4)

F4 (48) = T, which is made up of 1 component points $V_0 = \pm e_j$
and 4 component elements $V_+ = Vp$ & $V_- = Vm$
$\oplus$ T' = Tp = $\sqrt{2}$ V123 from {V1, V2, V3},

which are made up of 2 component vectors $\pm e_0 \pm e_i$ or 2 component spinor / anti − spinors $\pm e_i \pm e_j$)

These are shown in symbolic & rounded numeric forms w/Norms.

6) Apply σ scaling factor to T & V123 as (T, 0) ⊕ (0, T) ⊕ (V_1, V_3) ⊕ (V_2, V_1) ⊕ (V_3, V_2) (eq .5)

Where (A, B) = A + φB (vs . from E8 -> H4 folding) .
This creates 72 of 120 H4 φ scaled elements of 72 = 2×T (24) + T' (24) or 96 if you add the 24 from (-T') .
This is not the 240 E8 -> H4 + H4φ as suggested but ...
using the predefined 120 Icosians (I, I) = I + φI it does trivially create a folded E8 -> H4 + H4φ set.

These are shown in symbolic & rounded numeric forms w/Norms.

7) Define the 12 vertex Icosahedron set (IcosSet) using a specific combinations of {p,t1,t2}

Define the base Icosahedron set functions in string form (for formatting output).

Show the numeric and a final reduced symbolic form in terms of φ only as well as verify the octonion Norm=1.

In[7695]:=

```
rndOct /@ IcosSet // MatrixForm
octSym /@ % // MatrixForm
```

Out[7695]//MatrixForm=

$$\begin{pmatrix} 1.618 + 1.\, e_1 - 0.618\, e_3 \\ 1.618 - 1.\, e_1 + 0.618\, e_3 \\ 0. + 1.\, e_1 - 0.618\, e_2 - 1.618\, e_3 \\ 0.618 + 1.\, e_1 - 1.618\, e_2 \\ 1.618 + 0.618\, e_1 - 1.\, e_2 \\ 0. + 1.618\, e_1 - 1.\, e_2 - 0.618\, e_3 \\ 1. + 1.618\, e_1 + 0.618\, e_2 \\ 0.618 + 1.618\, e_1 - 1.\, e_3 \\ 1. + 1.618\, e_1 - 0.618\, e_2 \\ 1. - 1.618\, e_2 - 0.618\, e_3 \\ 0.618 + 1.\, e_2 + 1.618\, e_3 \\ 1. + 0.618\, e_1 - 1.618\, e_3 \end{pmatrix}$$

Out[7696]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\varphi} + 1.\, e_1 - \varphi\, e_3 \\ \frac{1}{\varphi} - 1.\, e_1 + \varphi\, e_3 \\ 0. + 1.\, e_1 - \varphi\, e_2 - \frac{e_3}{\varphi} \\ \varphi + 1.\, e_1 - \frac{e_2}{\varphi} \\ \frac{1}{\varphi} + \varphi\, e_1 - 1.\, e_2 \\ 0. + \frac{e_1}{\varphi} - 1.\, e_2 - \varphi\, e_3 \\ 1. + \frac{e_1}{\varphi} + \varphi\, e_2 \\ \varphi + \frac{e_1}{\varphi} - 1.\, e_3 \\ 1. + \frac{e_1}{\varphi} - \varphi\, e_2 \\ 1. - \frac{e_2}{\varphi} - \varphi\, e_3 \\ \varphi + 1.\, e_2 + \frac{e_3}{\varphi} \\ 1. + \varphi\, e_1 - \frac{e_3}{\varphi} \end{pmatrix}$$

8) Find the 48 p ∈ I where $p^5 = \pm 1$ and show $\overline{p}^1 == \pm p^4$ && $\overline{p}^4 == \pm p$, $\overline{p}^2 == p^3$ && $\overline{p}^3 == p^2$ (eq.8)

Show $\overline{p}^1 == \pm p^4$ && $\overline{p}^4 == \pm p$, $\overline{p}^2 == p^3$ && $\overline{p}^3 == p^2$ when $\overline{p}^5 == \pm p^5 == \pm \overline{p}^0 == \pm p^0 == \pm 1.$

Trim the ±1 from p ∈ I where $p^5 = \pm 1$ (which fail the power conjugate test).

In[7701]:=

```
(* Position in I *)
drop2I = Select[out, #[[2]] == 2. || #[[2]] == -2. &][[All, 1]]
```

Out[7701]=

```
{18, 103}
```

In[7702]:=

```
(* Position in p50 *)
drop2p = Flatten[Position[p50, #] & /@ drop2I, 1]
```

Out[7702]=

```
{{7}, {44}}
```

In[7703]:=

```
p48 = Delete[p50, drop2p]
Length@%
```

Out[7703]=

```
{2, 3, 4, 8, 12, 15, 19, 23, 26, 30, 32, 36, 37, 38, 39, 40, 41, 42, 47, 49, 52, 54, 57, 60, 61, 64,
 67, 69, 72, 74, 79, 80, 81, 82, 83, 84, 85, 89, 91, 95, 98, 102, 106, 109, 113, 117, 118, 119}
```

Out[7704]=

```
48
```

9) Create I (120) using T (24) and p^n, where p ∈ I && $p^5 = \pm 1$ with S (96) = I – T.

We are going to perform the I (120) process for each of 50 p ∈ I && $p^5 = \pm 1$ (including the two trivial cases) in order to demonstrate the issue between p50 vs. p48.

T (24) × 1 ($p^5 = \pm 1$) × 5 ($p^{n=0-4}$) = 120 permutations with no dups.

By definition $p^{j=0}$ T = 1. T, so the $p^{n=1-4}$ T will be I – T = S (96).

In[7705]:=

```
in1 = rndMat10@oct2List[#][[ ;; 4]] & /@ Sort@SgroupInt;
Join[
  {{"indx", "Generated", "Unique", Column[{"Naive", "Match", "Count", "__________"}, Center]}},
  Table[
   outTbl = TableForm@
     Table[Row@{inp, "  ", inT, "  ",
        MatrixForm[{#, out = rndOct[octPower[inp, #] ∘ inT],
            (* Powers p^n=1-4 or p^n=0-4 *)
            MemberQ[rndIcosO, out]}] & /@ Range[1, 4]},
      (* These are individual elements of the 50 p∈I where p^5=±1 *)
      {inp, rndIcosO[[{p50[[indx]]}]]},
      (* Only the first 24 are the original T Norm=1, the rest are scaled by σ *)
      {inT, TgroupInt[[ ;; 24]]}];

   outTblFlat = Flatten[outTbl[[1, All, All, 1, 5, All, 1, 2]], 2];
   Iout = DeleteDuplicates@outTblFlat;
   {indx,
    (* These Length's will be 96 when using Powers p^n=1-4 or 120 for p^n=0-4 *)
    Length@outTblFlat,
    Length@Iout,
    (* This will be True when using Powers p^n=1-4 on all but the 2 trivial p^5=±1 cases *)
    in2 = rndMat10@oct2List[#][[ ;; 4]] & /@ Sort@Iout;
    Count[in1[[#]] == in2[[#]] & /@ Range@Length@Iout, True]},
   {indx, Length@p50}]] // MatrixForm
```

Out[7706]//MatrixForm=

indx	Generated	Unique	Naive Match Count
1	96	96	72
2	96	96	56
3	96	96	57
4	96	96	44
5	96	96	43
6	96	96	48
7	96	24	0
8	96	96	96
9	96	96	96
10	96	96	96
11	96	96	96
12	96	96	96
13	96	96	96
14	96	96	96
15	96	96	96
16	96	96	96
17	96	96	96
18	96	96	96
19	96	96	96
20	96	96	66
21	96	96	65
22	96	96	60
23	96	96	48
24	96	96	54
25	96	96	51
26	96	96	51
27	96	96	54
28	96	96	48
29	96	96	60
30	96	96	65
31	96	96	66
32	96	96	96
33	96	96	96
34	96	96	96
35	96	96	96
36	96	96	96
37	96	96	96
38	96	96	96
39	96	96	96
40	96	96	96
41	96	96	96
42	96	96	96
43	96	96	96
44	96	24	0
45	96	96	48
46	96	96	43
47	96	96	44
48	96	96	57
49	96	96	56
50	96	96	72

Notice the positions 7 & 44 have 72 duplicates leaving only 24 vertices matching.
This coincides with drop2p = {{7}, {44}} obtained above .

10) Define and test the [p,q]:r function a prq[] (eq.6)

[p, q] : r is defined for combinations of inputs as elements or lists of equal length

In[8188]:=

```
prq[p_, r_, q_] := FullSimplify[p∘(r∘q)];
prq[p_List, r_, q_] := prq[p[[#]], r, q] & /@ Range@Length@p;
prq[p_, r_List, q_] := prq[p, r[[#]], q] & /@ Range@Length@r;
prq[p_, r_, q_List] := prq[p, r, q[[#]]] & /@ Range@Length@q;
prq[p_List, r_, q_List] := prq[p[[#]], r, q[[#]]] & /@ Range@Length@p;
prq[p_List, r_List, q_List] := prq[p[[#]], r[[#]], q[[#]]] & /@ Range@Length@p;
```

Show Power Conjugations $p_c^j = q \circ p^{j=0-4} \circ \overline{q}$ where p & q ∈ I using [p, q] : r → [q, $\overline{q}$] : p^j

11) Create 600 vertices (J) of the 120 Cell

(the dual of I (120) = 600 Cell) with $\sum_{i=0}^{4} p^i \circ (\overline{p}^{i\dagger} \circ I)$ (eq .15).

For reference, these are the “canonical” 600 Cell values based on permutations of Φ & φ based on the Cartesian coordinates of shown in Wikipedia.

Round the 120 Cell vertices for future comparison of values to be generated . These have a 4D Norm= $\sqrt{8}$.

In[7714]:=

```
rndCell120 = Sort@rndMat10[Cell120 /. {φ → (√5 - 1) / 2., Φ → (√5 + 1) / 2.}];
rndScaledCell120 = rndMat10[rndCell120 / √8.];
Length@%
```

Out[7716]=

```
600
```

In[7717]:=

```
(* Show the Norms are 2√2 =2.828...` *)
Union[rndMat10[Norm@# & /@ rndCell120]] == {2.828`}
```

Out[7717]=

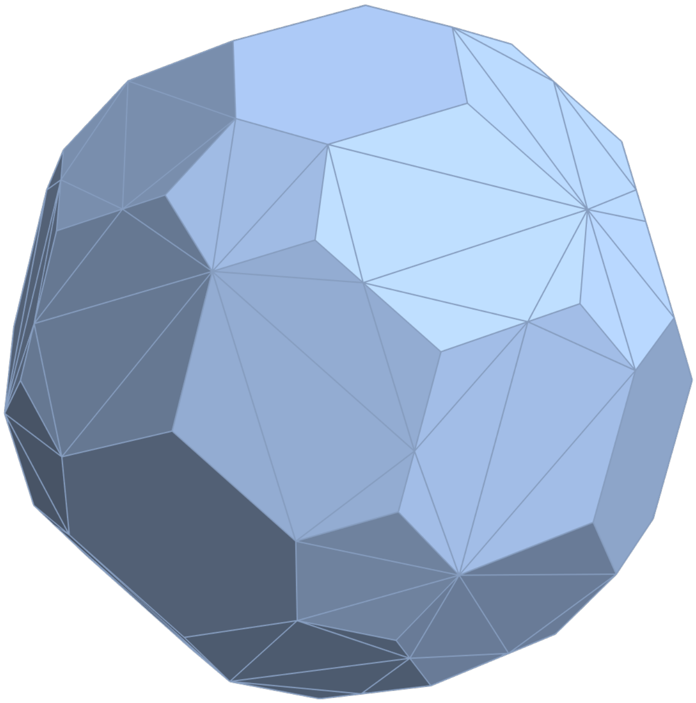
```
True
```

Visualize the canonical 120 Cell vertices.

In[7718]:=

```
ConvexHullMesh@rndCell120⟦All, 2 ;;⟧
```

Out[7718]=



From the 2007 paper, validate eq. 15 $\sum_{a=0}^{4} \alpha^a \circ (c \circ \beta^a) == \sum_{a=0}^{4} \alpha^a \circ (\overline{\alpha}^{\dagger a} \circ c)$

In[7719]:=

```
Join[{{"a", "α^a∘(c∘β^a)", "α^a∘(ᾱ^a†∘c)", "Match?"}},
  Table[{a,
    o1 = FullSimplify@rndMat@oct2List[prq[octPower[α, a], c, octPower[β, a]]];
    octonion@o1,
    o2 = FullSimplify@rndMat@oct2List[prq[octPower[α, a], octPower[switchστ@α*, a], c]];
    octonion@o2,
    rndMat10@Abs[o1 - o2] == Array[0. &, 8]},
   {a, 0, 4}]] // MatrixForm
```

Out[7719]//MatrixForm=

a	$\alpha^a \circ (c \circ \beta^a)$	$\alpha^a \circ (\overline{\alpha}^{a\dagger} \circ c)$	Match?
0	$0. + 0.7071 e_2 - 0.7071 e_3$	$0. + 0.7071 e_2 - 0.7071 e_3$	True
1	$0.7906 - 0.5721 e_2 - 0.2185 e_3$	$0.7906 - 0.572 e_2 - 0.2185 e_3$	True
2	$-0.7906 - 0.5721 e_2 - 0.2185 e_3$	$-0.7905 - 0.5721 e_2 - 0.2185 e_3$	True
3	$0. + 0.7906 e_1 + 0.2185 e_2 + 0.5721 e_3$	$0. + 0.7905 e_1 + 0.2185 e_2 + 0.572 e_3$	True
4	$0. - 0.7906 e_1 + 0.2185 e_2 + 0.5721 e_3$	$0. - 0.7905 e_1 + 0.2185 e_2 + 0.572 e_3$	True

From the 2007 paper, attempt to validate 120 cell {5, 3, 3} $\sum_{a=0}^{4} \bigoplus I \circ (\alpha^a \circ (\overline{\alpha}^{\dagger a} \circ c))$

In[7720]:=

```
out1 = Flatten[Join[{{{"a", "I∘(α^a∘(ᾱ^†a∘c))", "octNorm",
      Column[{"Naive", "Canonical", "Cell120", "Match", "__________"}, Center]}}},
    Table[{a,
      o1 = FullSimplify@rndMat@oct2List[inI∘prq[octPower[α, a], octPower[switchστ@α*, a], c]];
      octonion@o1,
      round10@octNorm@o1,
      (* Determine if the vertex matches one of the rounded and scaled canonical values *)
      MemberQ[rndScaledCell120, rndMat10@o1⟦;; 4⟧]},
     {a, 0, 4},
     {inI, rndIcosO}]], 1];
out1 // MatrixForm
Length@out1 - 1
```

Out[7721]//MatrixForm=

a	I∘(α^a∘(ᾱ^†a∘c))	octNorm	Naive Canonical Cell120 Match __________
0	$0. + 0.7071 e_1 - 0.7071 e_2$	1.	True
0	$-0.3536 + 0.3536 e_1 - 0.7905 e_2 - 0.3536 e_3$	1.	False
0	$-0.2185 + 0.9256 e_1 - 0.2185 e_2 + 0.2185 e_3$	1.	True
0	$0.572 + 0.572 e_1 - 0.572 e_2 - 0.1351 e_3$	1.	True
0	$0. + 0.7071 e_1 - 0.7071 e_3$	1.	True
0	$-0.572 + 0.572 e_1 - 0.572 e_2 + 0.1351 e_3$	1.	True
0	$0.2185 + 0.2185 e_1 - 0.9256 e_2 - 0.2185 e_3$	1.	True
0	$-0.3536 + 0.3536 e_1 - 0.3536 e_2 - 0.7905 e_3$	1.	False
0	$-0.7071 - 0.7071 e_2$	1.	True
0	$-0.2185 - 0.2185 e_1 - 0.9256 e_2 - 0.2185 e_3$	1.	True
0	$0.3536 + 0.7905 e_1 - 0.3536 e_2 + 0.3536 e_3$	1.	False
0	$-0.2185 + 0.9256 e_1 + 0.2185 e_2 - 0.2185 e_3$	1.	True
0	$-0.572 + 0.572 e_1 - 0.1351 e_2 + 0.572 e_3$	1.	True
0	$0. + 0.7071 e_1 + 0.7071 e_3$	1.	True
0	$0.572 + 0.572 e_1 - 0.1351 e_2 - 0.572 e_3$	1.	True
0	$0.7905 + 0.3536 e_1 - 0.3536 e_2 + 0.3536 e_3$	1.	False
0	$0.7071 - 0.7071 e_2$	1.	True
0	$0. - 0.7071 e_2 + 0.7071 e_3$	1.	True
0	$0.3536 + 0.3536 e_1 - 0.3536 e_2 + 0.7905 e_3$	1.	False
0	$0.7071 + 0.7071 e_1$	1.	True
0	$0.2185 + 0.9256 e_1 + 0.2185 e_2 + 0.2185 e_3$	1.	True
0	$0.572 + 0.1351 e_1 + 0.572 e_2 + 0.572 e_3$	1.	True
0	$0.3536 + 0.3536 e_1 + 0.7905 e_2 - 0.3536 e_3$	1.	False
0	$0.9256 + 0.2185 e_1 + 0.2185 e_2 + 0.2185 e_3$	1.	True
0	$0.1351 + 0.572 e_1 + 0.572 e_2 + 0.572 e_3$	1.	True
0	$0.2185 - 0.2185 e_1 - 0.9256 e_2 + 0.2185 e_3$	1.	True
0	$0.572 + 0.1351 e_1 - 0.572 e_2 - 0.572 e_3$	1.	True
0	$0. - 0.7071 e_2 - 0.7071 e_3$	1.	True
0	$0.3536 - 0.7905 e_1 - 0.3536 e_2 - 0.3536 e_3$	1.	False
0	$0.2185 - 0.2185 e_1 + 0.2185 e_2 - 0.9256 e_3$	1.	True
0	$0.7905 - 0.3536 e_1 - 0.3536 e_2 - 0.3536 e_3$	1.	False
0	$-0.572 + 0.1351 e_1 - 0.572 e_2 + 0.572 e_3$	1.	True
0	$-0.7905 + 0.3536 e_1 - 0.3536 e_2 - 0.3536 e_3$	1.	False
0	$-0.7071 + 0.7071 e_1$	1.	True
0	$-0.9256 - 0.2185 e_1 + 0.2185 e_2 + 0.2185 e_3$	1.	True
0	$-0.572 + 0.1351 e_1 + 0.572 e_2 - 0.572 e_3$	1.	True
0	$0.3536 + 0.3536 e_1 - 0.7905 e_2 + 0.3536 e_3$	1.	False
0	$-0.2185 + 0.2185 e_1 - 0.9256 e_2 + 0.2185 e_3$	1.	True
0	$-0.1351 + 0.572 e_1 - 0.572 e_2 + 0.572 e_3$	1.	True

0	$0.1351 - 0.572\, e_1 - 0.572\, e_2 + 0.572\, e_3$	1.	True
0	$0.2185 - 0.2185\, e_1 - 0.2185\, e_2 + 0.9256\, e_3$	1.	True
0	$-0.3536 - 0.3536\, e_1 - 0.3536\, e_2 + 0.7905\, e_3$	1.	False
0	$0.1351 + 0.572\, e_1 - 0.572\, e_2 - 0.572\, e_3$	1.	True
0	$0.2185 + 0.9256\, e_1 - 0.2185\, e_2 - 0.2185\, e_3$	1.	True
0	$-0.3536 + 0.7905\, e_1 - 0.3536\, e_2 - 0.3536\, e_3$	1.	False
0	$-0.572 + 0.572\, e_1 + 0.1351\, e_2 - 0.572\, e_3$	1.	True
0	$-0.9256 + 0.2185\, e_1 - 0.2185\, e_2 + 0.2185\, e_3$	1.	True
0	$0.2185 + 0.2185\, e_1 - 0.2185\, e_2 - 0.9256\, e_3$	1.	True
0	$0.3536 - 0.3536\, e_1 - 0.7905\, e_2 - 0.3536\, e_3$	1.	False
0	$-0.7071 - 0.7071\, e_3$	1.	True
0	$-0.2185 - 0.2185\, e_1 - 0.2185\, e_2 - 0.9256\, e_3$	1.	True
0	$-0.572 - 0.572\, e_1 - 0.572\, e_2 - 0.1351\, e_3$	1.	True
0	$0.3536 + 0.7905\, e_1 + 0.3536\, e_2 - 0.3536\, e_3$	1.	False
0	$0.572 + 0.572\, e_1 + 0.1351\, e_2 + 0.572\, e_3$	1.	True
0	$-0.572 + 0.572\, e_1 + 0.572\, e_2 - 0.1351\, e_3$	1.	True
0	$0. + 0.7071\, e_1 + 0.7071\, e_2$	1.	True
0	$-0.3536 + 0.3536\, e_1 + 0.3536\, e_2 + 0.7905\, e_3$	1.	False
0	$0.7905 + 0.3536\, e_1 + 0.3536\, e_2 - 0.3536\, e_3$	1.	False
0	$0.7071 - 0.7071\, e_3$	1.	True
0	$0.9256 - 0.2185\, e_1 - 0.2185\, e_2 + 0.2185\, e_3$	1.	True
0	$-0.9256 + 0.2185\, e_1 + 0.2185\, e_2 - 0.2185\, e_3$	1.	True
0	$-0.7071 + 0.7071\, e_3$	1.	True
0	$-0.7905 - 0.3536\, e_1 - 0.3536\, e_2 + 0.3536\, e_3$	1.	False
0	$0.3536 - 0.3536\, e_1 - 0.3536\, e_2 - 0.7905\, e_3$	1.	False
0	$0. - 0.7071\, e_1 - 0.7071\, e_2$	1.	True
0	$0.572 - 0.572\, e_1 - 0.572\, e_2 + 0.1351\, e_3$	1.	True
0	$-0.572 - 0.572\, e_1 - 0.1351\, e_2 - 0.572\, e_3$	1.	True
0	$-0.3536 - 0.7905\, e_1 - 0.3536\, e_2 + 0.3536\, e_3$	1.	False
0	$0.572 + 0.572\, e_1 + 0.572\, e_2 + 0.1351\, e_3$	1.	True
0	$0.2185 + 0.2185\, e_1 + 0.2185\, e_2 + 0.9256\, e_3$	1.	True
0	$0.7071 + 0.7071\, e_3$	1.	True
0	$-0.3536 + 0.3536\, e_1 + 0.7905\, e_2 + 0.3536\, e_3$	1.	False
0	$-0.2185 - 0.2185\, e_1 + 0.2185\, e_2 + 0.9256\, e_3$	1.	True
0	$0.9256 - 0.2185\, e_1 + 0.2185\, e_2 - 0.2185\, e_3$	1.	True
0	$0.572 - 0.572\, e_1 - 0.1351\, e_2 + 0.572\, e_3$	1.	True
0	$0.3536 - 0.7905\, e_1 + 0.3536\, e_2 + 0.3536\, e_3$	1.	False
0	$-0.2185 - 0.9256\, e_1 + 0.2185\, e_2 + 0.2185\, e_3$	1.	True
0	$-0.1351 - 0.572\, e_1 + 0.572\, e_2 + 0.572\, e_3$	1.	True
0	$0.3536 + 0.3536\, e_1 + 0.3536\, e_2 - 0.7905\, e_3$	1.	False
0	$-0.2185 + 0.2185\, e_1 + 0.2185\, e_2 - 0.9256\, e_3$	1.	True
0	$-0.1351 + 0.572\, e_1 + 0.572\, e_2 - 0.572\, e_3$	1.	True
0	$0.1351 - 0.572\, e_1 + 0.572\, e_2 - 0.572\, e_3$	1.	True
0	$0.2185 - 0.2185\, e_1 + 0.9256\, e_2 - 0.2185\, e_3$	1.	True
0	$-0.3536 - 0.3536\, e_1 + 0.7905\, e_2 - 0.3536\, e_3$	1.	False
0	$0.572 - 0.1351\, e_1 - 0.572\, e_2 + 0.572\, e_3$	1.	True
0	$0.9256 + 0.2185\, e_1 - 0.2185\, e_2 - 0.2185\, e_3$	1.	True
0	$0.7071 - 0.7071\, e_1$	1.	True
0	$0.7905 - 0.3536\, e_1 + 0.3536\, e_2 + 0.3536\, e_3$	1.	False
0	$0.572 - 0.1351\, e_1 + 0.572\, e_2 - 0.572\, e_3$	1.	True
0	$-0.7905 + 0.3536\, e_1 + 0.3536\, e_2 + 0.3536\, e_3$	1.	False
0	$-0.2185 + 0.2185\, e_1 - 0.2185\, e_2 + 0.9256\, e_3$	1.	True
0	$-0.3536 + 0.7905\, e_1 + 0.3536\, e_2 + 0.3536\, e_3$	1.	False
0	$0. + 0.7071\, e_2 + 0.7071\, e_3$	1.	True
0	$-0.572 - 0.1351\, e_1 + 0.572\, e_2 + 0.572\, e_3$	1.	True
0	$-0.2185 + 0.2185\, e_1 + 0.9256\, e_2 - 0.2185\, e_3$	1.	True
0	$-0.1351 - 0.572\, e_1 - 0.572\, e_2 - 0.572\, e_3$	1.	True
0	$-0.9256 - 0.2185\, e_1 - 0.2185\, e_2 - 0.2185\, e_3$	1.	True
0	$-0.3536 - 0.3536\, e_1 - 0.7905\, e_2 + 0.3536\, e_3$	1.	False
0	[illegible] 0.572 [illegible] $0.1351\, e_1$ [illegible] $0.572\, e_2$ [illegible] $0.572\, e_3$	1.	True

0	$-0.572 - 0.1351 e_1 - 0.572 e_2 - 0.572 e_3$	1.	True
0	$-0.2185 - 0.9256 e_1 - 0.2185 e_2 - 0.2185 e_3$	1.	True
0	$-0.7071 - 0.7071 e_1$	1.	True
0	$-0.3536 - 0.3536 e_1 + 0.3536 e_2 - 0.7905 e_3$	1.	False
0	$0. + 0.7071 e_2 - 0.7071 e_3$	1.	True
0	$-0.7071 + 0.7071 e_2$	1.	True
0	$-0.7905 - 0.3536 e_1 + 0.3536 e_2 - 0.3536 e_3$	1.	False
0	$-0.572 - 0.572 e_1 + 0.1351 e_2 + 0.572 e_3$	1.	True
0	$0. - 0.7071 e_1 - 0.7071 e_3$	1.	True
0	$0.572 - 0.572 e_1 + 0.1351 e_2 - 0.572 e_3$	1.	True
0	$0.2185 - 0.9256 e_1 - 0.2185 e_2 + 0.2185 e_3$	1.	True
0	$-0.3536 - 0.7905 e_1 + 0.3536 e_2 - 0.3536 e_3$	1.	False
0	$0.2185 + 0.2185 e_1 + 0.9256 e_2 + 0.2185 e_3$	1.	True
0	$0.7071 + 0.7071 e_2$	1.	True
0	$0.3536 - 0.3536 e_1 + 0.3536 e_2 + 0.7905 e_3$	1.	False
0	$-0.2185 - 0.2185 e_1 + 0.9256 e_2 + 0.2185 e_3$	1.	True
0	$0.572 - 0.572 e_1 + 0.572 e_2 - 0.1351 e_3$	1.	True
0	$0. - 0.7071 e_1 + 0.7071 e_3$	1.	True
0	$-0.572 - 0.572 e_1 + 0.572 e_2 + 0.1351 e_3$	1.	True
0	$0.2185 - 0.9256 e_1 + 0.2185 e_2 - 0.2185 e_3$	1.	True
0	$0.3536 - 0.3536 e_1 + 0.7905 e_2 + 0.3536 e_3$	1.	False
0	$0. - 0.7071 e_1 + 0.7071 e_2$	1.	True
1	$-0.7906 - 0.572 e_1 - 0.2185 e_2$	1.	True
1	$-0.3535 - 0.9256 e_1 + 0.135 e_3$	1.	True
1	$-0.7071 - 0.3535 e_1 - 0.2185 e_2 - 0.572 e_3$	1.	True
1	$-0.7071 - 0.2185 e_1 - 0.5721 e_2 + 0.3535 e_3$	1.	True
1	$0. - 0.572 e_1 - 0.7906 e_2 - 0.2185 e_3$	1.	True
1	$-0.5721 - 0.7071 e_1 + 0.2185 e_2 - 0.3535 e_3$	1.	True
1	$-0.572 - 0.572 e_1 - 0.135 e_2 + 0.572 e_3$	1.	True
1	$0.135 - 0.9256 e_1 - 0.3535 e_2$	1.	True
1	$-0.2185 - 0.7906 e_1 + 0.572 e_2$	1.	True
1	$-0.2185 - 0.7071 e_1 + 0.3535 e_2 + 0.572 e_3$	1.	True
1	$-0.9256 - 0.3535 e_2 - 0.135 e_3$	1.	True
1	$-0.2185 - 0.3535 e_1 - 0.572 e_2 - 0.7071 e_3$	1.	True
1	$-0.5721 - 0.2185 e_1 + 0.3535 e_2 - 0.7071 e_3$	1.	True
1	$-0.7906 + 0.2185 e_1 - 0.572 e_3$	1.	True
1	$-0.2185 - 0.2185 e_1 - 0.9256 e_2 + 0.2185 e_3$	1.	True
1	$-0.7905 + 0.3535 e_1 - 0.3535 e_2 + 0.3535 e_3$	1.	False
1	$-0.572 - 0.2185 e_2 + 0.7906 e_3$	1.	True
1	$-0.7906 + 0.572 e_2 + 0.2185 e_3$	1.	True
1	$-0.9256 + 0.3535 e_1 + 0.135 e_2$	1.	True
1	$-0.572 + 0.2185 e_1 - 0.7906 e_2$	1.	True
1	$-0.572 + 0.135 e_1 - 0.572 e_2 - 0.572 e_3$	1.	True
1	$-0.2185 + 0.9256 e_1 - 0.2185 e_2 - 0.2185 e_3$	1.	True
1	$0.3535 + 0.3535 e_1 - 0.7905 e_2 - 0.3535 e_3$	1.	False
1	$-0.3535 + 0.7071 e_1 - 0.572 e_2 + 0.2185 e_3$	1.	True
1	$-0.3535 + 0.5721 e_1 - 0.2185 e_2 - 0.7071 e_3$	1.	True
1	$-0.572 - 0.2185 e_1 + 0.3535 e_2 + 0.7071 e_3$	1.	True
1	$-0.2185 - 0.3535 e_1 - 0.5721 e_2 + 0.7071 e_3$	1.	True
1	$0. - 0.7906 e_1 - 0.2185 e_2 + 0.572 e_3$	1.	True
1	$0.3535 + 0.135 e_2 + 0.9256 e_3$	1.	True
1	$0.7071 - 0.2185 e_1 - 0.572 e_2 + 0.3535 e_3$	1.	True
1	$0. + 0.135 e_1 - 0.3535 e_2 + 0.9256 e_3$	1.	True
1	$-0.5721 - 0.3535 e_1 + 0.7071 e_2 - 0.2185 e_3$	1.	True
1	$0. - 0.9256 e_1 + 0.135 e_2 - 0.3535 e_3$	1.	True
1	$-0.2185 - 0.572 e_1 - 0.7906 e_3$	1.	True
1	$0.3535 - 0.2185 e_1 + 0.7071 e_2 - 0.572 e_3$	1.	True
1	$0.7071 - 0.3535 e_1 - 0.2185 e_2 - 0.5721 e_3$	1.	True
1	$-0.9256 - 0.135 e_1 + 0.3535 e_3$	1.	True
1	$-0.7071 - 0.572 e_1 + 0.3535 e_2 + 0.2185 e_3$	1.	True

1	$-0.9256 - 0.2185\, e_1 + 0.2185\, e_2 - 0.2185\, e_3$	1.	True
1	$-0.3535 + 0.2185\, e_1 + 0.7071\, e_2 + 0.5721\, e_3$	1.	True
1	$-0.572 + 0.572\, e_1 + 0.572\, e_2 + 0.135\, e_3$	1.	True
1	$-0.3535 + 0.135\, e_1 + 0.9256\, e_2$	1.	True
1	$-0.3535 - 0.7071\, e_1 - 0.5721\, e_2 + 0.2185\, e_3$	1.	True
1	$-0.572 - 0.3535\, e_1 - 0.7071\, e_2 - 0.2185\, e_3$	1.	True
1	$-0.3535 - 0.7905\, e_1 - 0.3535\, e_2 - 0.3535\, e_3$	1.	False
1	$0.2185 - 0.7071\, e_1 - 0.3535\, e_2 - 0.5721\, e_3$	1.	True
1	$-0.135 - 0.572\, e_1 + 0.572\, e_2 - 0.572\, e_3$	1.	True
1	$0.2185 - 0.572\, e_1 - 0.7071\, e_2 + 0.3535\, e_3$	1.	True
1	$-0.135 - 0.3535\, e_1 + 0.9256\, e_3$	1.	True
1	$0.572 - 0.7906\, e_1 - 0.2185\, e_3$	1.	True
1	$0.572 - 0.7071\, e_1 - 0.2185\, e_2 + 0.3535\, e_3$	1.	True
1	$0.2185 - 0.5721\, e_1 + 0.7071\, e_2 + 0.3535\, e_3$	1.	True
1	$-0.135 - 0.9256\, e_2 - 0.3535\, e_3$	1.	True
1	$-0.7071 + 0.5721\, e_1 - 0.3535\, e_2 - 0.2185\, e_3$	1.	True
1	$0.2185 - 0.2185\, e_1 - 0.2185\, e_2 - 0.9256\, e_3$	1.	True
1	$0. + 0.2185\, e_1 - 0.572\, e_2 - 0.7906\, e_3$	1.	True
1	$-0.3535 + 0.3535\, e_1 + 0.3535\, e_2 - 0.7905\, e_3$	1.	False
1	$0. + 0.3535\, e_1 - 0.9256\, e_2 + 0.135\, e_3$	1.	True
1	$0.2185 - 0.7906\, e_2 + 0.572\, e_3$	1.	True
1	$-0.3535 + 0.572\, e_1 - 0.2185\, e_2 + 0.7071\, e_3$	1.	True
1	$0.3535 - 0.572\, e_1 + 0.2185\, e_2 - 0.7071\, e_3$	1.	True
1	$-0.2185 + 0.7906\, e_2 - 0.572\, e_3$	1.	True
1	$0. - 0.3535\, e_1 + 0.9256\, e_2 - 0.135\, e_3$	1.	True
1	$0.3535 - 0.3535\, e_1 - 0.3535\, e_2 + 0.7905\, e_3$	1.	False
1	$0. - 0.2185\, e_1 + 0.572\, e_2 + 0.7906\, e_3$	1.	True
1	$-0.2185 + 0.2185\, e_1 + 0.2185\, e_2 + 0.9256\, e_3$	1.	True
1	$0.7071 - 0.5721\, e_1 + 0.3535\, e_2 + 0.2185\, e_3$	1.	True
1	$0.135 + 0.9256\, e_2 + 0.3535\, e_3$	1.	True
1	$-0.2185 + 0.5721\, e_1 - 0.7071\, e_2 - 0.3535\, e_3$	1.	True
1	$-0.572 + 0.7071\, e_1 + 0.2185\, e_2 - 0.3535\, e_3$	1.	True
1	$-0.572 + 0.7906\, e_1 + 0.2185\, e_3$	1.	True
1	$0.135 + 0.3535\, e_1 - 0.9256\, e_3$	1.	True
1	$-0.2185 + 0.572\, e_1 + 0.7071\, e_2 - 0.3535\, e_3$	1.	True
1	$0.135 + 0.572\, e_1 - 0.572\, e_2 + 0.572\, e_3$	1.	True
1	$-0.2185 + 0.7071\, e_1 + 0.3535\, e_2 + 0.5721\, e_3$	1.	True
1	$0.3535 + 0.7905\, e_1 + 0.3535\, e_2 + 0.3535\, e_3$	1.	False
1	$0.572 + 0.3535\, e_1 + 0.7071\, e_2 + 0.2185\, e_3$	1.	True
1	$0.3535 + 0.7071\, e_1 + 0.5721\, e_2 - 0.2185\, e_3$	1.	True
1	$0.3535 - 0.135\, e_1 - 0.9256\, e_2$	1.	True
1	$0.572 - 0.572\, e_1 - 0.572\, e_2 - 0.135\, e_3$	1.	True
1	$0.3535 - 0.2185\, e_1 - 0.7071\, e_2 - 0.5721\, e_3$	1.	True
1	$0.9256 + 0.2185\, e_1 - 0.2185\, e_2 + 0.2185\, e_3$	1.	True
1	$0.7071 + 0.572\, e_1 - 0.3535\, e_2 - 0.2185\, e_3$	1.	True
1	$0.9256 + 0.135\, e_1 - 0.3535\, e_3$	1.	True
1	$-0.7071 + 0.3535\, e_1 + 0.2185\, e_2 + 0.5721\, e_3$	1.	True
1	$-0.3535 + 0.2185\, e_1 - 0.7071\, e_2 + 0.572\, e_3$	1.	True
1	$0.2185 + 0.572\, e_1 + 0.7906\, e_3$	1.	True
1	$0. + 0.9256\, e_1 - 0.135\, e_2 + 0.3535\, e_3$	1.	True
1	$0.5721 + 0.3535\, e_1 - 0.7071\, e_2 + 0.2185\, e_3$	1.	True
1	$0. - 0.135\, e_1 + 0.3535\, e_2 - 0.9256\, e_3$	1.	True
1	$-0.7071 + 0.2185\, e_1 + 0.572\, e_2 - 0.3535\, e_3$	1.	True
1	$-0.3535 - 0.135\, e_2 - 0.9256\, e_3$	1.	True
1	$0. + 0.7906\, e_1 + 0.2185\, e_2 - 0.572\, e_3$	1.	True
1	$0.2185 + 0.3535\, e_1 + 0.5721\, e_2 - 0.7071\, e_3$	1.	True
1	$0.572 + 0.2185\, e_1 - 0.3535\, e_2 - 0.7071\, e_3$	1.	True
1	$0.3535 - 0.5721\, e_1 + 0.2185\, e_2 + 0.7071\, e_3$	1.	True
1	$0.3535 - 0.7071\, e_1 + 0.572\, e_2 - 0.2185\, e_3$	1.	True

1	$-0.3535 - 0.3535\,e_1 + 0.7905\,e_2 + 0.3535\,e_3$	1.	False
1	$0.2185 - 0.9256\,e_1 + 0.2185\,e_2 + 0.2185\,e_3$	1.	True
1	$0.572 - 0.135\,e_1 + 0.572\,e_2 + 0.572\,e_3$	1.	True
1	$0.572 - 0.2185\,e_1 + 0.7906\,e_2$	1.	True
1	$0.9256 - 0.3535\,e_1 - 0.135\,e_2$	1.	True
1	$0.7906 - 0.572\,e_2 - 0.2185\,e_3$	1.	True
1	$0.572 + 0.2185\,e_2 - 0.7906\,e_3$	1.	True
1	$0.7905 - 0.3535\,e_1 + 0.3535\,e_2 - 0.3535\,e_3$	1.	False
1	$0.2185 + 0.2185\,e_1 + 0.9256\,e_2 - 0.2185\,e_3$	1.	True
1	$0.7906 - 0.2185\,e_1 + 0.572\,e_3$	1.	True
1	$0.5721 + 0.2185\,e_1 - 0.3535\,e_2 + 0.7071\,e_3$	1.	True
1	$0.2185 + 0.3535\,e_1 + 0.572\,e_2 + 0.7071\,e_3$	1.	True
1	$0.9256 + 0.3535\,e_2 + 0.135\,e_3$	1.	True
1	$0.2185 + 0.7071\,e_1 - 0.3535\,e_2 - 0.572\,e_3$	1.	True
1	$0.2185 + 0.7906\,e_1 - 0.572\,e_2$	1.	True
1	$-0.135 + 0.9256\,e_1 + 0.3535\,e_2$	1.	True
1	$0.572 + 0.572\,e_1 + 0.135\,e_2 - 0.572\,e_3$	1.	True
1	$0.5721 + 0.7071\,e_1 - 0.2185\,e_2 + 0.3535\,e_3$	1.	True
1	$0. + 0.572\,e_1 + 0.7906\,e_2 + 0.2185\,e_3$	1.	True
1	$0.7071 + 0.2185\,e_1 + 0.5721\,e_2 - 0.3535\,e_3$	1.	True
1	$0.7071 + 0.3535\,e_1 + 0.2185\,e_2 + 0.572\,e_3$	1.	True
1	$0.3535 + 0.9256\,e_1 - 0.135\,e_3$	1.	True
1	$0.7906 + 0.572\,e_1 + 0.2185\,e_2$	1.	True
2	$0. + 0.2185\,e_1 + 0.572\,e_2 + 0.7905\,e_3$	1.	False
2	$0.135 + 0.3535\,e_1 + 0.9256\,e_3$	1.	True
2	$-0.2185 - 0.3535\,e_1 + 0.572\,e_2 + 0.7071\,e_3$	1.	True
2	$-0.2185 + 0.572\,e_1 + 0.7071\,e_2 + 0.3536\,e_3$	1.	True
2	$-0.7905 + 0.2185\,e_1 + 0.572\,e_3$	1.	False
2	$0.2185 - 0.2185\,e_1 + 0.2185\,e_2 + 0.9256\,e_3$	1.	True
2	$0.2185 + 0.707\,e_1 + 0.3535\,e_2 + 0.572\,e_3$	1.	True
2	$-0.3535 + 0.3535\,e_1 - 0.3535\,e_2 + 0.7905\,e_3$	1.	False
2	$0.572 - 0.2185\,e_2 + 0.7905\,e_3$	1.	False
2	$0.572 + 0.572\,e_1 - 0.135\,e_2 + 0.572\,e_3$	1.	True
2	$-0.135 + 0.9256\,e_2 + 0.3535\,e_3$	1.	True
2	$-0.7071 - 0.3535\,e_1 + 0.2185\,e_2 + 0.572\,e_3$	1.	True
2	$0.2185 - 0.7071\,e_1 + 0.3535\,e_2 + 0.572\,e_3$	1.	True
2	$0. - 0.572\,e_1 + 0.7905\,e_2 + 0.2185\,e_3$	1.	False
2	$-0.7071 + 0.572\,e_1 + 0.3535\,e_2 + 0.2185\,e_3$	1.	True
2	$0. + 0.3535\,e_1 + 0.9256\,e_2 - 0.135\,e_3$	1.	True
2	$0.2185 + 0.7905\,e_1 + 0.572\,e_2$	1.	False
2	$0.7905 + 0.5721\,e_2 + 0.2185\,e_3$	1.	False
2	$0.3535 - 0.135\,e_1 + 0.9256\,e_2$	1.	True
2	$-0.5721 + 0.2185\,e_1 + 0.7905\,e_2$	1.	False
2	$-0.572 - 0.3535\,e_1 + 0.707\,e_2 + 0.2185\,e_3$	1.	True
2	$-0.2185 - 0.3535\,e_1 + 0.572\,e_2 - 0.7071\,e_3$	1.	True
2	$-0.9256 - 0.135\,e_1 - 0.3535\,e_3$	1.	True
2	$-0.3535 + 0.2185\,e_1 + 0.7071\,e_2 - 0.572\,e_3$	1.	True
2	$-0.3536 - 0.7071\,e_1 + 0.572\,e_2 - 0.2185\,e_3$	1.	True
2	$0.7071 + 0.572\,e_1 + 0.3535\,e_2 + 0.2185\,e_3$	1.	True
2	$-0.2185 + 0.9256\,e_1 + 0.2185\,e_2 + 0.2185\,e_3$	1.	True
2	$0. + 0.7905\,e_1 - 0.2185\,e_2 + 0.5721\,e_3$	1.	False
2	$0.3535 + 0.7905\,e_1 - 0.3535\,e_2 - 0.3535\,e_3$	1.	False
2	$-0.572 + 0.572\,e_1 - 0.572\,e_2 - 0.135\,e_3$	1.	True
2	$0. + 0.9256\,e_1 + 0.135\,e_2 - 0.3535\,e_3$	1.	True
2	$0.7071 - 0.3536\,e_1 + 0.2185\,e_2 + 0.572\,e_3$	1.	True
2	$0. - 0.135\,e_1 - 0.3535\,e_2 + 0.9256\,e_3$	1.	True
2	$-0.2185 - 0.5721\,e_1 + 0.7905\,e_3$	1.	False
2	$0.3535 - 0.7071\,e_1 - 0.572\,e_2 + 0.2185\,e_3$	1.	True
2	$-0.572 - 0.3535\,e_1 - 0.7071\,e_2 + 0.2185\,e_3$	1.	True

2	$0.3535 + 0.3535 e_1 + 0.7905 e_2 + 0.3535 e_3$	1.	False
2	$0.572 + 0.2185 e_1 + 0.3535 e_2 + 0.7071 e_3$	1.	True
2	$0.3535 - 0.2185 e_1 + 0.7071 e_2 + 0.572 e_3$	1.	True
2	$0.9256 + 0.2185 e_1 + 0.2185 e_2 - 0.2185 e_3$	1.	True
2	$0.707 - 0.2185 e_1 + 0.572 e_2 - 0.3535 e_3$	1.	True
2	$0.9256 - 0.3535 e_1 + 0.135 e_2$	1.	True
2	$-0.3535 + 0.572 e_1 + 0.2185 e_2 + 0.7071 e_3$	1.	True
2	$-0.572 + 0.135 e_1 + 0.572 e_2 + 0.572 e_3$	1.	True
2	$-0.3535 + 0.135 e_2 + 0.9256 e_3$	1.	True
2	$-0.572 - 0.2185 e_1 - 0.3536 e_2 + 0.7071 e_3$	1.	True
2	$0.3535 - 0.572 e_1 - 0.2185 e_2 + 0.707 e_3$	1.	True
2	$-0.572 + 0.7071 e_1 - 0.2185 e_2 + 0.3535 e_3$	1.	True
2	$0.3535 + 0.9256 e_1 + 0.135 e_3$	1.	True
2	$-0.2185 - 0.7905 e_2 + 0.572 e_3$	1.	False
2	$-0.2185 + 0.572 e_1 - 0.7071 e_2 + 0.3535 e_3$	1.	True
2	$0.7071 + 0.2185 e_1 - 0.572 e_2 + 0.3535 e_3$	1.	True
2	$-0.9256 + 0.3535 e_2 + 0.135 e_3$	1.	True
2	$-0.2185 - 0.2185 e_1 + 0.9256 e_2 - 0.2185 e_3$	1.	True
2	$-0.572 - 0.7071 e_1 - 0.2185 e_2 + 0.3535 e_3$	1.	True
2	$-0.7905 - 0.572 e_1 + 0.2185 e_2$	1.	False
2	$0.135 - 0.9256 e_1 + 0.3535 e_2$	1.	True
2	$-0.7905 + 0.3535 e_1 + 0.3535 e_2 - 0.3535 e_3$	1.	False
2	$-0.572 + 0.7905 e_1 - 0.2185 e_3$	1.	False
2	$0.135 + 0.572 e_1 + 0.572 e_2 - 0.572 e_3$	1.	True
2	$-0.135 - 0.572 e_1 - 0.572 e_2 + 0.572 e_3$	1.	True
2	$0.572 - 0.7905 e_1 + 0.2185 e_3$	1.	False
2	$0.7905 - 0.3535 e_1 - 0.3535 e_2 + 0.3535 e_3$	1.	False
2	$-0.135 + 0.9256 e_1 - 0.3535 e_2$	1.	True
2	$0.7905 + 0.572 e_1 - 0.2185 e_2$	1.	False
2	$0.572 + 0.7071 e_1 + 0.2185 e_2 - 0.3535 e_3$	1.	True
2	$0.2185 + 0.2185 e_1 - 0.9256 e_2 + 0.2185 e_3$	1.	True
2	$0.9256 - 0.3535 e_2 - 0.135 e_3$	1.	True
2	$-0.7071 - 0.2185 e_1 + 0.572 e_2 - 0.3535 e_3$	1.	True
2	$0.2185 - 0.572 e_1 + 0.7071 e_2 - 0.3535 e_3$	1.	True
2	$0.2185 + 0.7905 e_2 - 0.572 e_3$	1.	False
2	$-0.3535 - 0.9256 e_1 - 0.135 e_3$	1.	True
2	$0.572 - 0.7071 e_1 + 0.2185 e_2 - 0.3535 e_3$	1.	True
2	$-0.3535 + 0.572 e_1 + 0.2185 e_2 - 0.707 e_3$	1.	True
2	$0.572 + 0.2185 e_1 + 0.3536 e_2 - 0.7071 e_3$	1.	True
2	$0.3535 - 0.135 e_2 - 0.9256 e_3$	1.	True
2	$0.572 - 0.135 e_1 - 0.572 e_2 - 0.572 e_3$	1.	True
2	$0.3535 - 0.572 e_1 - 0.2185 e_2 - 0.7071 e_3$	1.	True
2	$-0.9256 + 0.3535 e_1 - 0.135 e_2$	1.	True
2	$-0.707 + 0.2185 e_1 - 0.572 e_2 + 0.3535 e_3$	1.	True
2	$-0.9256 - 0.2185 e_1 - 0.2185 e_2 + 0.2185 e_3$	1.	True
2	$-0.3535 + 0.2185 e_1 - 0.7071 e_2 - 0.572 e_3$	1.	True
2	$-0.572 - 0.2185 e_1 - 0.3535 e_2 - 0.7071 e_3$	1.	True
2	$-0.3535 - 0.3535 e_1 - 0.7905 e_2 - 0.3535 e_3$	1.	False
2	$0.572 + 0.3535 e_1 + 0.7071 e_2 - 0.2185 e_3$	1.	True
2	$-0.3535 + 0.7071 e_1 + 0.572 e_2 - 0.2185 e_3$	1.	True
2	$0.2185 + 0.5721 e_1 - 0.7905 e_3$	1.	False
2	$0. + 0.135 e_1 + 0.3535 e_2 - 0.9256 e_3$	1.	True
2	$-0.7071 + 0.3536 e_1 - 0.2185 e_2 - 0.572 e_3$	1.	True
2	$0. - 0.9256 e_1 - 0.135 e_2 + 0.3535 e_3$	1.	True
2	$0.572 - 0.572 e_1 + 0.572 e_2 + 0.135 e_3$	1.	True
2	$-0.3535 - 0.7905 e_1 + 0.3535 e_2 + 0.3535 e_3$	1.	False
2	$0. - 0.7905 e_1 + 0.2185 e_2 - 0.5721 e_3$	1.	False
2	$0.2185 - 0.9256 e_1 - 0.2185 e_2 - 0.2185 e_3$	1.	True
2	$-0.7071 - 0.572 e_1 - 0.3535 e_2 - 0.2185 e_3$	1.	True
2	$0.3536 + 0.7071 e_1 - 0.572 e_2 + 0.2185 e_3$	1.	True

2	[illegible]	1.	True
2	$0.3535 - 0.2185\, e_1 - 0.7071\, e_2 + 0.572\, e_3$	1.	True
2	$0.9256 + 0.135\, e_1 + 0.3535\, e_3$	1.	True
2	$0.2185 + 0.3535\, e_1 - 0.572\, e_2 + 0.7071\, e_3$	1.	True
2	$0.572 + 0.3535\, e_1 - 0.707\, e_2 - 0.2185\, e_3$	1.	True
2	$0.5721 - 0.2185\, e_1 - 0.7905\, e_2$	1.	False
2	$-0.3535 + 0.135\, e_1 - 0.9256\, e_2$	1.	True
2	$-0.7905 - 0.5721\, e_2 - 0.2185\, e_3$	1.	False
2	$-0.2185 - 0.7905\, e_1 - 0.572\, e_2$	1.	False
2	$0. - 0.3535\, e_1 - 0.9256\, e_2 + 0.135\, e_3$	1.	True
2	$0.7071 - 0.572\, e_1 - 0.3535\, e_2 - 0.2185\, e_3$	1.	True
2	$0. + 0.572\, e_1 - 0.7905\, e_2 - 0.2185\, e_3$	1.	False
2	$-0.2185 + 0.7071\, e_1 - 0.3535\, e_2 - 0.572\, e_3$	1.	True
2	$0.7071 + 0.3535\, e_1 - 0.2185\, e_2 - 0.572\, e_3$	1.	True
2	$0.135 - 0.9256\, e_2 - 0.3535\, e_3$	1.	True
2	$-0.572 - 0.572\, e_1 + 0.135\, e_2 - 0.572\, e_3$	1.	True
2	$-0.572 + 0.2185\, e_2 - 0.7905\, e_3$	1.	False
2	$0.3535 - 0.3535\, e_1 + 0.3535\, e_2 - 0.7905\, e_3$	1.	False
2	$-0.2185 - 0.707\, e_1 - 0.3535\, e_2 - 0.572\, e_3$	1.	True
2	$-0.2185 + 0.2185\, e_1 - 0.2185\, e_2 - 0.9256\, e_3$	1.	True
2	$0.7905 - 0.2185\, e_1 - 0.572\, e_3$	1.	False
2	$0.2185 - 0.572\, e_1 - 0.7071\, e_2 - 0.3536\, e_3$	1.	True
2	$0.2185 + 0.3535\, e_1 - 0.572\, e_2 - 0.7071\, e_3$	1.	True
2	$-0.135 - 0.3535\, e_1 - 0.9256\, e_3$	1.	True
2	$0. - 0.2185\, e_1 - 0.572\, e_2 - 0.7905\, e_3$	1.	False
3	$0.7905 - 0.572\, e_1 - 0.2185\, e_2$	1.	False
3	$0.9256 - 0.135\, e_1 - 0.3535\, e_3$	1.	True
3	$0.572 - 0.3535\, e_1 - 0.707\, e_2 + 0.2185\, e_3$	1.	True
3	$0.572 - 0.7071\, e_1 + 0.2185\, e_2 + 0.3535\, e_3$	1.	True
3	$0.7905 + 0.2185\, e_1 + 0.572\, e_3$	1.	False
3	$0.7071 - 0.2185\, e_1 - 0.572\, e_2 - 0.3535\, e_3$	1.	True
3	$0.707 - 0.572\, e_1 + 0.3535\, e_2 - 0.2185\, e_3$	1.	True
3	$0.9255 + 0.3535\, e_1 + 0.135\, e_2$	1.	True
3	$0.572 - 0.2185\, e_2 - 0.7905\, e_3$	1.	False
3	$0.572 - 0.2185\, e_1 + 0.3535\, e_2 - 0.707\, e_3$	1.	True
3	$0.3535 - 0.7905\, e_1 - 0.3535\, e_2 + 0.3535\, e_3$	1.	False
3	$0.572 + 0.135\, e_1 - 0.572\, e_2 + 0.572\, e_3$	1.	True
3	$0.2185 - 0.2185\, e_1 - 0.9255\, e_2 - 0.2185\, e_3$	1.	True
3	$0. - 0.572\, e_1 - 0.7905\, e_2 + 0.2185\, e_3$	1.	False
3	$0.572 - 0.2185\, e_1 + 0.3535\, e_2 + 0.707\, e_3$	1.	True
3	$0. - 0.9256\, e_1 + 0.135\, e_2 + 0.3535\, e_3$	1.	True
3	$0.2185 - 0.7905\, e_1 + 0.572\, e_2$	1.	False
3	$0. - 0.7905\, e_1 - 0.2185\, e_2 - 0.572\, e_3$	1.	False
3	$-0.135 - 0.9256\, e_1 - 0.3535\, e_2$	1.	True
3	$0.2185 - 0.572\, e_1 + 0.7905\, e_3$	1.	False
3	$0.2185 - 0.3535\, e_1 - 0.572\, e_2 + 0.707\, e_3$	1.	True
3	$-0.707 - 0.3535\, e_1 - 0.2185\, e_2 + 0.572\, e_3$	1.	True
3	$-0.135 + 0.3535\, e_1 + 0.9256\, e_3$	1.	True
3	$-0.3535 - 0.572\, e_1 + 0.2185\, e_2 + 0.707\, e_3$	1.	True
3	$-0.3535 - 0.2185\, e_1 - 0.7071\, e_2 + 0.572\, e_3$	1.	True
3	$0.2185 - 0.707\, e_1 + 0.3535\, e_2 - 0.572\, e_3$	1.	True
3	$0.572 - 0.3535\, e_1 + 0.7071\, e_2 + 0.2185\, e_3$	1.	True
3	$0.7905 + 0.572\, e_2 - 0.2185\, e_3$	1.	False
3	$-0.135 + 0.9255\, e_2 - 0.3535\, e_3$	1.	True
3	$0.2185 + 0.572\, e_1 + 0.707\, e_2 + 0.3535\, e_3$	1.	True
3	$0. - 0.3535\, e_1 + 0.9256\, e_2 + 0.135\, e_3$	1.	True
3	$0.2185 - 0.3535\, e_1 - 0.572\, e_2 - 0.7071\, e_3$	1.	True
3	$0.7905 + 0.3535\, e_1 - 0.3535\, e_2 - 0.3535\, e_3$	1.	False
3	$0.572 + 0.2185\, e_1 - 0.7905\, e_2$	1.	False
3	$-0.135 + 0.572\, e_1 - 0.572\, e_2 - 0.572\, e_3$	1.	True

3	$0.2185 + 0.9255\, e_1 - 0.2185\, e_2 + 0.2185\, e_3$	1.	True
3	$0.3535 - 0.9255\, e_1 - 0.135\, e_3$	1.	True
3	$0.572 - 0.572\, e_1 - 0.135\, e_2 - 0.572\, e_3$	1.	True
3	$0.3535 - 0.707\, e_1 - 0.572\, e_2 - 0.2185\, e_3$	1.	True
3	$-0.3535 - 0.572\, e_1 + 0.2185\, e_2 - 0.7071\, e_3$	1.	True
3	$-0.572 - 0.707\, e_1 - 0.2185\, e_2 - 0.3535\, e_3$	1.	True
3	$-0.3535 - 0.3535\, e_1 - 0.3535\, e_2 - 0.7905\, e_3$	1.	False
3	$0.9255 - 0.2185\, e_1 + 0.2185\, e_2 + 0.2185\, e_3$	1.	True
3	$0.707 - 0.3535\, e_1 - 0.2185\, e_2 + 0.572\, e_3$	1.	True
3	$0.9256 - 0.3535\, e_2 + 0.135\, e_3$	1.	True
3	$0.7071 + 0.572\, e_1 - 0.3535\, e_2 + 0.2185\, e_3$	1.	True
3	$0.3535 + 0.2185\, e_1 - 0.707\, e_2 - 0.572\, e_3$	1.	True
3	$0.707 + 0.2185\, e_1 + 0.572\, e_2 + 0.3535\, e_3$	1.	True
3	$0.3535 - 0.3535\, e_1 + 0.7905\, e_2 - 0.3535\, e_3$	1.	False
3	$0.572 + 0.7905\, e_1 - 0.2185\, e_3$	1.	False
3	$0.572 + 0.572\, e_1 + 0.572\, e_2 - 0.135\, e_3$	1.	True
3	$0.2185 + 0.2185\, e_1 + 0.2185\, e_2 - 0.9255\, e_3$	1.	True
3	$0.3535 - 0.135\, e_2 + 0.9256\, e_3$	1.	True
3	$-0.2185 - 0.7071\, e_1 - 0.3535\, e_2 + 0.572\, e_3$	1.	True
3	$0.2185 + 0.572\, e_1 - 0.707\, e_2 + 0.3535\, e_3$	1.	True
3	$0. + 0.2185\, e_1 - 0.572\, e_2 + 0.7905\, e_3$	1.	False
3	$-0.3535 - 0.135\, e_1 - 0.9256\, e_2$	1.	True
3	$0. - 0.135\, e_1 + 0.3535\, e_2 + 0.9255\, e_3$	1.	True
3	$0.2185 + 0.7905\, e_2 + 0.572\, e_3$	1.	False
3	$-0.3535 - 0.707\, e_1 + 0.572\, e_2 + 0.2185\, e_3$	1.	True
3	$0.3535 + 0.707\, e_1 - 0.572\, e_2 - 0.2185\, e_3$	1.	True
3	$-0.2185 - 0.7905\, e_2 - 0.572\, e_3$	1.	False
3	$0. + 0.135\, e_1 - 0.3535\, e_2 - 0.9255\, e_3$	1.	True
3	$0.3535 + 0.135\, e_1 + 0.9256\, e_2$	1.	True
3	$0. - 0.2185\, e_1 + 0.572\, e_2 - 0.7905\, e_3$	1.	False
3	$-0.2185 - 0.572\, e_1 + 0.707\, e_2 - 0.3535\, e_3$	1.	True
3	$0.2185 + 0.7071\, e_1 + 0.3535\, e_2 - 0.572\, e_3$	1.	True
3	$-0.3535 + 0.135\, e_2 - 0.9256\, e_3$	1.	True
3	$-0.2185 - 0.2185\, e_1 - 0.2185\, e_2 + 0.9255\, e_3$	1.	True
3	$-0.572 - 0.572\, e_1 - 0.572\, e_2 + 0.135\, e_3$	1.	True
3	$-0.572 - 0.7905\, e_1 + 0.2185\, e_3$	1.	False
3	$-0.3535 + 0.3535\, e_1 - 0.7905\, e_2 + 0.3535\, e_3$	1.	False
3	$-0.707 - 0.2185\, e_1 - 0.572\, e_2 - 0.3535\, e_3$	1.	True
3	$-0.3535 - 0.2185\, e_1 + 0.707\, e_2 + 0.572\, e_3$	1.	True
3	$-0.7071 - 0.572\, e_1 + 0.3535\, e_2 - 0.2185\, e_3$	1.	True
3	$-0.9256 + 0.3535\, e_2 - 0.135\, e_3$	1.	True
3	$-0.707 + 0.3535\, e_1 + 0.2185\, e_2 - 0.572\, e_3$	1.	True
3	$-0.9255 + 0.2185\, e_1 - 0.2185\, e_2 - 0.2185\, e_3$	1.	True
3	$0.3535 + 0.3535\, e_1 + 0.3535\, e_2 + 0.7905\, e_3$	1.	False
3	$0.572 + 0.707\, e_1 + 0.2185\, e_2 + 0.3535\, e_3$	1.	True
3	$0.3535 + 0.572\, e_1 - 0.2185\, e_2 + 0.7071\, e_3$	1.	True
3	$-0.3535 + 0.707\, e_1 + 0.572\, e_2 + 0.2185\, e_3$	1.	True
3	$-0.572 + 0.572\, e_1 + 0.135\, e_2 + 0.572\, e_3$	1.	True
3	$-0.3535 + 0.9255\, e_1 + 0.135\, e_3$	1.	True
3	$-0.2185 - 0.9255\, e_1 + 0.2185\, e_2 - 0.2185\, e_3$	1.	True
3	$0.135 - 0.572\, e_1 + 0.572\, e_2 + 0.572\, e_3$	1.	True
3	$-0.572 - 0.2185\, e_1 + 0.7905\, e_2$	1.	False
3	$-0.7905 - 0.3535\, e_1 + 0.3535\, e_2 + 0.3535\, e_3$	1.	False
3	$-0.2185 + 0.3535\, e_1 + 0.572\, e_2 + 0.7071\, e_3$	1.	True
3	$0. + 0.3535\, e_1 - 0.9256\, e_2 - 0.135\, e_3$	1.	True
3	$-0.2185 - 0.572\, e_1 - 0.707\, e_2 - 0.3535\, e_3$	1.	True
3	$0.135 - 0.9255\, e_2 + 0.3535\, e_3$	1.	True
3	$-0.7905 - 0.572\, e_2 + 0.2185\, e_3$	1.	False
3	$-0.572 + 0.3535\, e_1 - 0.7071\, e_2 - 0.2185\, e_3$	1.	True

3	$-0.2185 + 0.707 e_1 - 0.3535 e_2 + 0.572 e_3$	1.	True
3	$0.3535 + 0.2185 e_1 + 0.7071 e_2 - 0.572 e_3$	1.	True
3	$0.3535 + 0.572 e_1 - 0.2185 e_2 - 0.707 e_3$	1.	True
3	$0.135 - 0.3535 e_1 - 0.9256 e_3$	1.	True
3	$0.707 + 0.3535 e_1 + 0.2185 e_2 - 0.572 e_3$	1.	True
3	$-0.2185 + 0.3535 e_1 + 0.572 e_2 - 0.707 e_3$	1.	True
3	$-0.2185 + 0.572 e_1 - 0.7905 e_3$	1.	False
3	$0.135 + 0.9256 e_1 + 0.3535 e_2$	1.	True
3	$0. + 0.7905 e_1 + 0.2185 e_2 + 0.572 e_3$	1.	False
3	$-0.2185 + 0.7905 e_1 - 0.572 e_2$	1.	False
3	$0. + 0.9256 e_1 - 0.135 e_2 - 0.3535 e_3$	1.	True
3	$-0.572 + 0.2185 e_1 - 0.3535 e_2 - 0.707 e_3$	1.	True
3	$0. + 0.572 e_1 + 0.7905 e_2 - 0.2185 e_3$	1.	False
3	$-0.2185 + 0.2185 e_1 + 0.9255 e_2 + 0.2185 e_3$	1.	True
3	$-0.572 - 0.135 e_1 + 0.572 e_2 - 0.572 e_3$	1.	True
3	$-0.3535 + 0.7905 e_1 + 0.3535 e_2 - 0.3535 e_3$	1.	False
3	$-0.572 + 0.2185 e_1 - 0.3535 e_2 + 0.707 e_3$	1.	True
3	$-0.572 + 0.2185 e_2 + 0.7905 e_3$	1.	False
3	$-0.9255 - 0.3535 e_1 - 0.135 e_2$	1.	True
3	$-0.707 + 0.572 e_1 - 0.3535 e_2 + 0.2185 e_3$	1.	True
3	$-0.7071 + 0.2185 e_1 + 0.572 e_2 + 0.3535 e_3$	1.	True
3	$-0.7905 - 0.2185 e_1 - 0.572 e_3$	1.	False
3	$-0.572 + 0.7071 e_1 - 0.2185 e_2 - 0.3535 e_3$	1.	True
3	$-0.572 + 0.3535 e_1 + 0.707 e_2 - 0.2185 e_3$	1.	True
3	$-0.9256 + 0.135 e_1 + 0.3535 e_3$	1.	True
3	$-0.7905 + 0.572 e_1 + 0.2185 e_2$	1.	False
4	$0. + 0.2185 e_1 + 0.572 e_2 - 0.7905 e_3$	1.	False
4	$-0.3535 + 0.3535 e_1 + 0.7905 e_2 - 0.3535 e_3$	1.	False
4	$0.572 + 0.135 e_1 + 0.572 e_2 - 0.572 e_3$	1.	True
4	$-0.2185 - 0.2185 e_1 + 0.2185 e_2 - 0.9255 e_3$	1.	True
4	$0. - 0.572 e_1 + 0.7905 e_2 - 0.2185 e_3$	1.	False
4	$0.2185 + 0.572 e_1 + 0.707 e_2 - 0.3535 e_3$	1.	True
4	$-0.572 + 0.2185 e_1 + 0.3535 e_2 - 0.707 e_3$	1.	True
4	$-0.3535 - 0.135 e_1 + 0.9255 e_2$	1.	True
4	$-0.2185 + 0.7905 e_1 + 0.572 e_2$	1.	False
4	$-0.707 + 0.572 e_1 + 0.3535 e_2 - 0.2185 e_3$	1.	True
4	$0.3535 + 0.135 e_2 - 0.9255 e_3$	1.	True
4	$0.572 - 0.3535 e_1 + 0.707 e_2 - 0.2185 e_3$	1.	True
4	$0.707 + 0.572 e_1 + 0.3535 e_2 - 0.2185 e_3$	1.	True
4	$0.7905 + 0.2185 e_1 - 0.572 e_3$	1.	False
4	$-0.2185 - 0.707 e_1 + 0.3535 e_2 - 0.572 e_3$	1.	True
4	$0. - 0.135 e_1 - 0.3535 e_2 - 0.9255 e_3$	1.	True
4	$-0.572 - 0.2185 e_2 - 0.7905 e_3$	1.	False
4	$0. + 0.7905 e_1 - 0.2185 e_2 - 0.572 e_3$	1.	False
4	$0.3535 + 0.3535 e_1 - 0.3535 e_2 - 0.7905 e_3$	1.	False
4	$0.2185 - 0.572 e_1 - 0.7905 e_3$	1.	False
4	$0.707 - 0.3535 e_1 + 0.2185 e_2 - 0.572 e_3$	1.	True
4	$0.572 - 0.3535 e_1 - 0.707 e_2 - 0.2185 e_3$	1.	True
4	$0.3535 - 0.9255 e_1 + 0.135 e_3$	1.	True
4	$0.135 - 0.572 e_1 - 0.572 e_2 - 0.572 e_3$	1.	True
4	$0.9255 - 0.2185 e_1 - 0.2185 e_2 - 0.2185 e_3$	1.	True
4	$-0.572 + 0.572 e_1 - 0.135 e_2 - 0.572 e_3$	1.	True
4	$-0.707 - 0.3535 e_1 + 0.2185 e_2 - 0.572 e_3$	1.	True
4	$-0.7905 + 0.572 e_2 - 0.2185 e_3$	1.	False
4	$-0.9255 - 0.3535 e_2 + 0.135 e_3$	1.	True
4	$-0.572 - 0.707 e_1 + 0.2185 e_2 + 0.3535 e_3$	1.	True
4	$-0.7905 - 0.3535 e_1 - 0.3535 e_2 - 0.3535 e_3$	1.	False
4	$0.2185 + 0.9255 e_1 + 0.2185 e_2 - 0.2185 e_3$	1.	True
4	$0. + 0.3535 e_1 + 0.9255 e_2 + 0.135 e_3$	1.	True
4	$0.572 + 0.2185 e_1 + 0.7905$ e[illegible]	1.	False

4	$0.572 + 0.2185\,e_1 + 0.7905\,e_2$	1.	False
4	$0.3535 + 0.572\,e_1 + 0.2185\,e_2 + 0.707\,e_3$	1.	True
4	$0.2185 - 0.3535\,e_1 + 0.572\,e_2 + 0.707\,e_3$	1.	True
4	$-0.135 + 0.3535\,e_1 - 0.9255\,e_3$	1.	True
4	$-0.2185 + 0.707\,e_1 + 0.3535\,e_2 - 0.572\,e_3$	1.	True
4	$0.3535 + 0.572\,e_1 + 0.2185\,e_2 - 0.707\,e_3$	1.	True
4	$-0.3535 + 0.707\,e_1 - 0.572\,e_2 - 0.2185\,e_3$	1.	True
4	$0.2185 + 0.572\,e_1 - 0.707\,e_2 - 0.3535\,e_3$	1.	True
4	$0.135 + 0.9255\,e_1 - 0.3535\,e_2$	1.	True
4	$-0.3535 - 0.2185\,e_1 + 0.707\,e_2 - 0.572\,e_3$	1.	True
4	$0.2185 - 0.3535\,e_1 + 0.572\,e_2 - 0.707\,e_3$	1.	True
4	$0.135 + 0.9255\,e_2 - 0.3535\,e_3$	1.	True
4	$0.2185 - 0.2185\,e_1 + 0.9255\,e_2 + 0.2185\,e_3$	1.	True
4	$0.3535 + 0.707\,e_1 + 0.572\,e_2 + 0.2185\,e_3$	1.	True
4	$-0.572 - 0.572\,e_1 + 0.572\,e_2 - 0.135\,e_3$	1.	True
4	$-0.9255 + 0.135\,e_1 - 0.3535\,e_3$	1.	True
4	$-0.2185 + 0.7905\,e_2 + 0.572\,e_3$	1.	False
4	$-0.707 - 0.2185\,e_1 + 0.572\,e_2 + 0.3535\,e_3$	1.	True
4	$-0.572 + 0.707\,e_1 + 0.2185\,e_2 + 0.3535\,e_3$	1.	True
4	$0.3535 - 0.7905\,e_1 + 0.3535\,e_2 - 0.3535\,e_3$	1.	False
4	$0.572 - 0.2185\,e_1 - 0.3535\,e_2 - 0.707\,e_3$	1.	True
4	$0.707 - 0.2185\,e_1 + 0.572\,e_2 + 0.3535\,e_3$	1.	True
4	$0.7905 - 0.572\,e_1 + 0.2185\,e_2$	1.	False
4	$0.9255 + 0.3535\,e_1 - 0.135\,e_2$	1.	True
4	$0. - 0.9255\,e_1 - 0.135\,e_2 - 0.3535\,e_3$	1.	True
4	$-0.572 - 0.7905\,e_1 - 0.2185\,e_3$	1.	False
4	$-0.3535 - 0.2185\,e_1 - 0.707\,e_2 - 0.572\,e_3$	1.	True
4	$0.3535 + 0.2185\,e_1 + 0.707\,e_2 + 0.572\,e_3$	1.	True
4	$0.572 + 0.7905\,e_1 + 0.2185\,e_3$	1.	False
4	$0. + 0.9255\,e_1 + 0.135\,e_2 + 0.3535\,e_3$	1.	True
4	$-0.9255 - 0.3535\,e_1 + 0.135\,e_2$	1.	True
4	$-0.7905 + 0.572\,e_1 - 0.2185\,e_2$	1.	False
4	$-0.707 + 0.2185\,e_1 - 0.572\,e_2 - 0.3535\,e_3$	1.	True
4	$-0.572 + 0.2185\,e_1 + 0.3535\,e_2 + 0.707\,e_3$	1.	True
4	$-0.3535 + 0.7905\,e_1 - 0.3535\,e_2 + 0.3535\,e_3$	1.	False
4	$0.572 - 0.707\,e_1 - 0.2185\,e_2 - 0.3535\,e_3$	1.	True
4	$0.707 + 0.2185\,e_1 - 0.572\,e_2 - 0.3535\,e_3$	1.	True
4	$0.2185 - 0.7905\,e_2 - 0.572\,e_3$	1.	False
4	$0.9255 - 0.135\,e_1 + 0.3535\,e_3$	1.	True
4	$0.572 + 0.572\,e_1 - 0.572\,e_2 + 0.135\,e_3$	1.	True
4	$-0.3535 - 0.707\,e_1 - 0.572\,e_2 - 0.2185\,e_3$	1.	True
4	$-0.2185 + 0.2185\,e_1 - 0.9255\,e_2 - 0.2185\,e_3$	1.	True
4	$-0.135 - 0.9255\,e_2 + 0.3535\,e_3$	1.	True
4	$-0.2185 + 0.3535\,e_1 - 0.572\,e_2 + 0.707\,e_3$	1.	True
4	$0.3535 + 0.2185\,e_1 - 0.707\,e_2 + 0.572\,e_3$	1.	True
4	$-0.135 - 0.9255\,e_1 + 0.3535\,e_2$	1.	True
4	$-0.2185 - 0.572\,e_1 + 0.707\,e_2 + 0.3535\,e_3$	1.	True
4	$0.3535 - 0.707\,e_1 + 0.572\,e_2 + 0.2185\,e_3$	1.	True
4	$-0.3535 - 0.572\,e_1 - 0.2185\,e_2 + 0.707\,e_3$	1.	True
4	$0.2185 - 0.707\,e_1 - 0.3535\,e_2 + 0.572\,e_3$	1.	True
4	$0.135 - 0.3535\,e_1 + 0.9255\,e_3$	1.	True
4	$-0.2185 + 0.3535\,e_1 - 0.572\,e_2 - 0.707\,e_3$	1.	True
4	$-0.3535 - 0.572\,e_1 - 0.2185\,e_2 - 0.707\,e_3$	1.	True
4	$-0.572 - 0.2185\,e_1 - 0.7905\,e_2$	1.	False
4	$0. - 0.3535\,e_1 - 0.9255\,e_2 - 0.135\,e_3$	1.	True
4	$-0.2185 - 0.9255\,e_1 - 0.2185\,e_2 + 0.2185\,e_3$	1.	True
4	$0.7905 + 0.3535\,e_1 + 0.3535\,e_2 + 0.3535\,e_3$	1.	False
4	$0.572 + 0.707\,e_1 - 0.2185\,e_2 - 0.3535\,e_3$	1.	True
4	$0.9255 + 0.3535\,e_2 - 0.135\,e_3$	1.	True
4	$0.7905 - 0.572\,e_2 + 0.2185\,e_3$	1.	False

4	$0.707 + 0.3535 e_1 - 0.2185 e_2 + 0.572 e_3$	1.	True
4	$0.572 - 0.572 e_1 + 0.135 e_2 + 0.572 e_3$	1.	True
4	$-0.9255 + 0.2185 e_1 + 0.2185 e_2 + 0.2185 e_3$	1.	True
4	$-0.135 + 0.572 e_1 + 0.572 e_2 + 0.572 e_3$	1.	True
4	$-0.3535 + 0.9255 e_1 - 0.135 e_3$	1.	True
4	$-0.572 + 0.3535 e_1 + 0.707 e_2 + 0.2185 e_3$	1.	True
4	$-0.707 + 0.3535 e_1 - 0.2185 e_2 + 0.572 e_3$	1.	True
4	$-0.2185 + 0.572 e_1 + 0.7905 e_3$	1.	False
4	$-0.3535 - 0.3535 e_1 + 0.3535 e_2 + 0.7905 e_3$	1.	False
4	$0. - 0.7905 e_1 + 0.2185 e_2 + 0.572 e_3$	1.	False
4	$0.572 + 0.2185 e_2 + 0.7905 e_3$	1.	False
4	$0. + 0.135 e_1 + 0.3535 e_2 + 0.9255 e_3$	1.	True
4	$0.2185 + 0.707 e_1 - 0.3535 e_2 + 0.572 e_3$	1.	True
4	$-0.7905 - 0.2185 e_1 + 0.572 e_3$	1.	False
4	$-0.707 - 0.572 e_1 - 0.3535 e_2 + 0.2185 e_3$	1.	True
4	$-0.572 + 0.3535 e_1 - 0.707 e_2 + 0.2185 e_3$	1.	True
4	$-0.3535 - 0.135 e_2 + 0.9255 e_3$	1.	True
4	$0.707 - 0.572 e_1 - 0.3535 e_2 + 0.2185 e_3$	1.	True
4	$0.2185 - 0.7905 e_1 - 0.572 e_2$	1.	False
4	$0.3535 + 0.135 e_1 - 0.9255 e_2$	1.	True
4	$0.572 - 0.2185 e_1 - 0.3535 e_2 + 0.707 e_3$	1.	True
4	$-0.2185 - 0.572 e_1 - 0.707 e_2 + 0.3535 e_3$	1.	True
4	$0. + 0.572 e_1 - 0.7905 e_2 + 0.2185 e_3$	1.	False
4	$0.2185 + 0.2185 e_1 - 0.2185 e_2 + 0.9255 e_3$	1.	True
4	$-0.572 - 0.135 e_1 - 0.572 e_2 + 0.572 e_3$	1.	True
4	$0.3535 - 0.3535 e_1 - 0.7905 e_2 + 0.3535 e_3$	1.	False
4	$0. - 0.2185 e_1 - 0.572 e_2 + 0.7905 e_3$	1.	False

Out[7722]=

600

In[7723]:=

```
cell120Gen1 = Sort[rndMat10[oct2List@#] & /@ out1⟦2 ;;, 2⟧];
cell120Gen1 // MatrixForm;
```

From the 2007 paper, attempt to validate 120 cell {5, 3, 3} $\sum_{a,b=0}^{4} \oplus \alpha^{a} \circ \left(T' \circ \beta^{b}\right)$

Show these are equivalent to each other.

A naive Sort of the 4 D quaternion (or actually 8 D octonion) vertices show mismatches.

Evaluating the Norm vs. the Abs difference being 0 still miss 36 vertices.

In[7731]:=

```
outSel = Select[out3, #⟦5⟧ == True &];
Length@outSel
(* Evaluating the Total Abs difference being within ±.01 of 0 *)
outSel = Select[out3, Total@#⟦4⟧ < .01 &]⟦All, 1⟧;
Length@outSel
(* Matches vs. mismatches *)
600 - %
```

Out[7732]=

454

Out[7734]=

564

Out[7735]=

36

The 36 mismatches are due to Sort errors caused by rounding errors in e_0

Match up the mismatching vertices by resorting them to find the nearest 4D Quaternion (or 8D octonion) distanced vertex, verifying they are all the same vertex within ±.01 of 0

Show that these also match the rounded and scaled canonical 120 Cell vertices.

The naive MemberQ matching,
464 of the 600 quaternion generated vertices match the canonical values.

In[7740]:=

```
Length@out1 - 1
Count[out1⟦All, 4⟧, True]
```

Out[7740]=

600

Out[7741]=

464

Match up the 136 unmatched generated 120 Cell with canonical 120 Cell vertices by resorting to find the nearest 4D Quaternion (or 8D octonion) distanced vertex, verifying they are all the same vertex within ±.01 of 0

Validate the 120 Cell with 2009 paper $\sum_{i,j=0}^{4} p^i \circ (T' \circ \overline{q}^j)$ (eq .15)

12) Create and visualize the 96 vertices of $S' = \sum_{i=1}^{4} p^i \circ (\overline{p}^{i\dagger} \circ T')$, along with $M = M1 \oplus M2$ where $M1 = \sum_{i=1}^{4} p^i \circ T'$ and $M2 = \sum_{i=1}^{4} \overline{p}^{i\dagger} \circ T'$.

Create $S' = \sum_{i=1}^{4} p^i \circ (\overline{p}^{i\dagger} \circ T')$

In[8192]:=

```
SPgroupInt = Flatten[Table[
    rndOct /@ prq[octPower[α, a], octPower[switchστ@α*, a], TPgroupInt⟦ ;; 24⟧],
    {a, 1, 4}], 1];
Length@SPgroupInt
SPgroupInt;
```

Out[8193]=

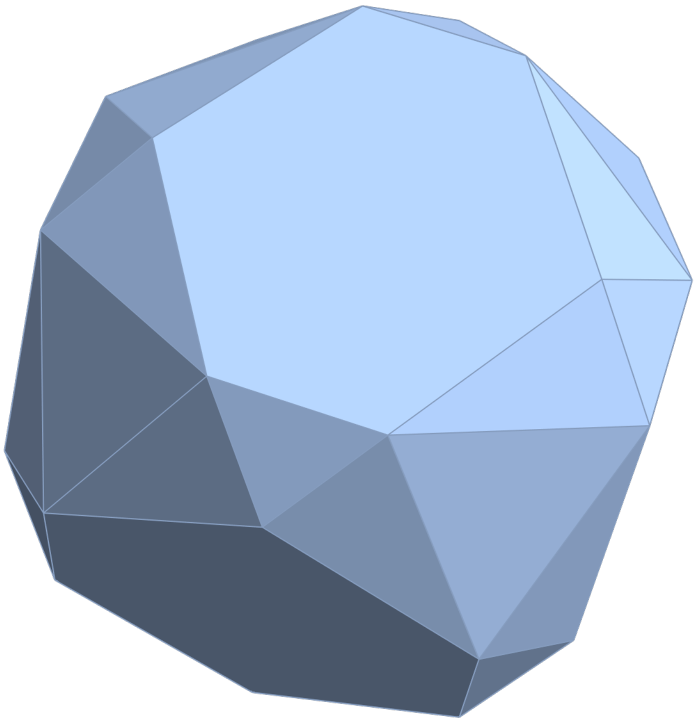
96

Visualize S'

In[8195]:=

```
ConvexHullMesh[oct2List[#]⟦;; 3⟧ & /@ SPgroupInt]
```

Out[8195]=



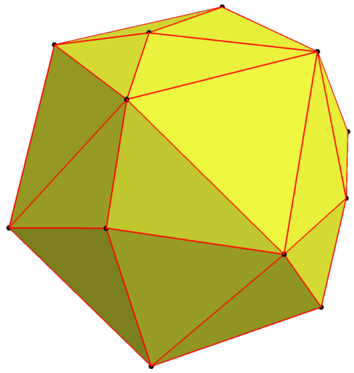
Tally and visualize S' 3D hull Norms using varying Opacity

In[8196]:=

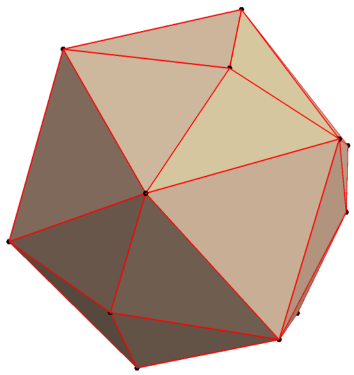
```
hulls3D[oct2List[#]⟦ ;; 3⟧ & /@ SPgroupInt]
```

Out[8196]=

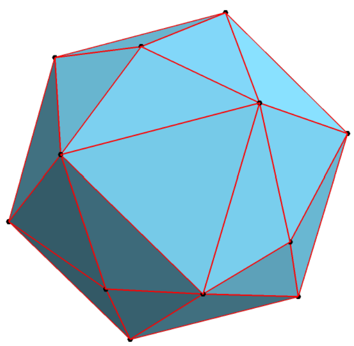
24   Verts of 3D Norm= 0.612



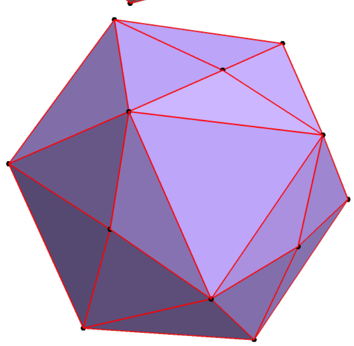
24   Verts of 3D Norm= 0.82

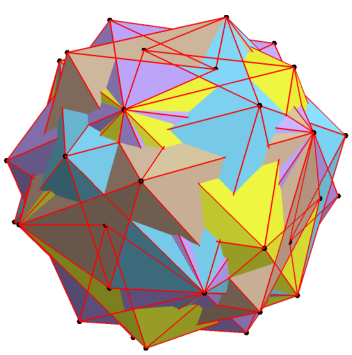


24   Verts of 3D Norm= 0.976



24   Verts of 3D Norm= 1.





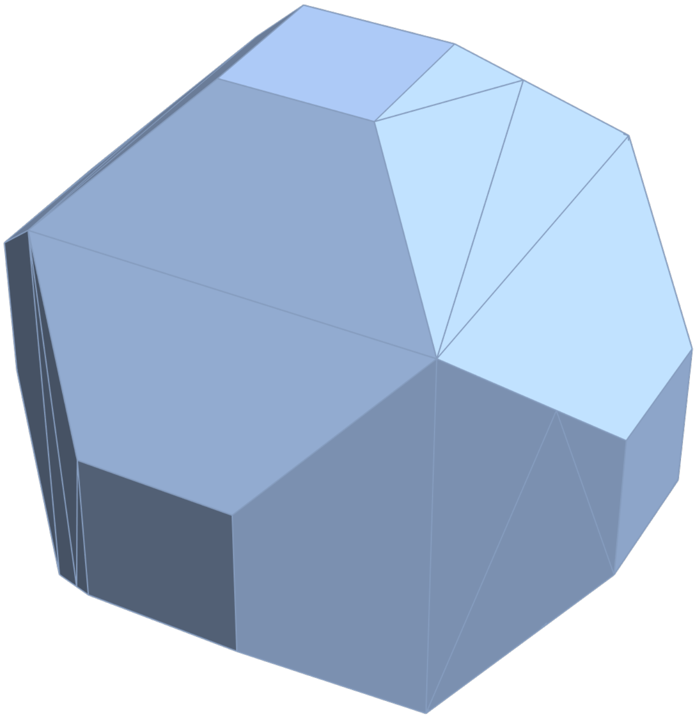
Create M1 $= \sum_{i=1}^{4} p^{i} \circ T'$

Visualize M1

In[7769]:=

```
ConvexHullMesh[oct2List[#]⟦ ;; 3⟧ & /@ M1groupInt]
```

Out[7769]=



Tally and visualize M1 3D hull Norms using varying Opacity

Create M2 $= \sum_{i=1}^{4} \overline{p}^{i\,\dagger} \circ T'$

Visualize M2

Visualize M(192)

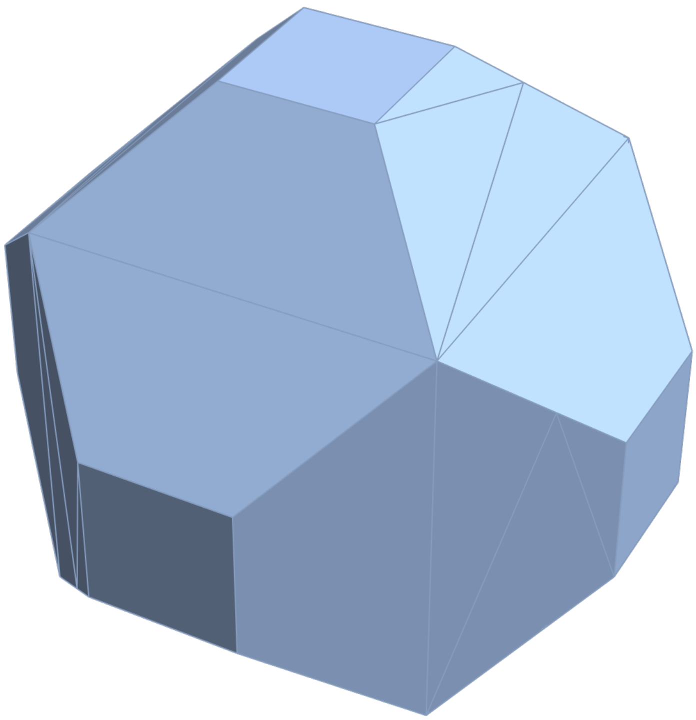
In[7787]:=

```
Length@Union@MgroupInt
ConvexHullMesh[oct2List[#]⟦ ;; 3⟧ & /@ MgroupInt]
```

Out[7787]=

192

Out[7788]=



Tally and visualize M(192) 3D hull Norms using varying Opacity

13) Visualize the 144 vertices of the dual snub 24 Cell $= T \oplus T' \oplus S' = \sum_{i=1}^{4} p^i \circ (\bar{p}^{i\dagger} \circ T')$.

Visualize $T \oplus T' \oplus S'$

In[7801]:=

```
dualSnub24Cell = oct2List[#]⟦ ;; 4⟧ & /@ Join[
    TgroupInt⟦ ;; 24⟧,
    TPgroupInt⟦ ;; 24⟧,
    SPgroupInt];
Length@%
```

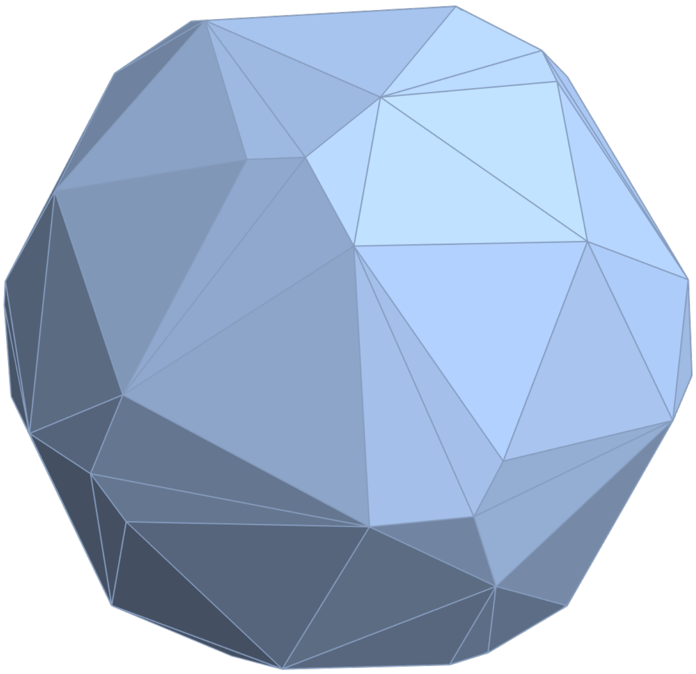
Out[7802]=

144

In[7803]:=

```
(* Visualize the dualSnub24Cell *)
ConvexHullMesh@dualSnub24Cell⟦All, ;; 3⟧
```

Out[7803]=



Tally and visualize dualSnub24Cell 3D hull Norms using varying Opacity

An alternate version of $S' = \sum_{i=1}^{4} p^i \circ \left(T' \circ \overline{p}^{i\,\dagger}\right)$