

```
In[*]:=
DeleteDuplicates[FullSimplify[Sort@Abs[ $\sqrt{8}$  #], Assumptions  $\rightarrow$  { $\varphi \in \text{Reals}$ ,  $\varphi > 0$ }] & /@ listJL $\Phi$ ]

Out[*]=
{ {0, 0, 0, 0, 0, 0, 2, 2}, {0, 0, 0, 0, 1, 1, 1,  $\sqrt{5}$ }, {0, 0, 0, 0,  $\frac{1}{\varphi}$ ,  $\frac{1}{\varphi}$ ,  $\frac{1}{\varphi}$ ,  $\varphi^2$ }, {0, 0, 0, 0,  $\frac{1}{\varphi^2}$ ,  $\varphi$ ,  $\varphi$ ,  $\varphi$ }, {0, 0, 0, 0, 0,  $\sqrt{5}$ ,  $\frac{1}{\varphi}$ ,  $\varphi$ }, {0, 0, 0, 0, 1, 2,  $\frac{1}{\varphi}$ ,  $\varphi$ }, {0, 0, 0, 0, 0, 1,  $\frac{1}{\varphi^2}$ ,  $\varphi^2$ }}
```

```
In[*]:=
JL $\Phi$  = octSimplify /@ Flatten@prq[prq[octPwr $\alpha$ L, TL, 1], octPwr $\alpha$ Lsw, cpL];
Length@%
JL $\Phi$ Rnd = rndOct /@ %%;
listJL $\Phi$  = FullSimplify[oct2List[#] & /@ %%, Assumptions  $\rightarrow$  { $\varphi \in \text{Reals}$ ,  $\varphi > 0$ }];
listJL $\Phi$ Rnd = oct2List /@ %%;
(**)
hulls3DPerms["listJL $\Phi$ ", False, , 1]
```

```
Out[*]=
600
```

Out[*]=

ListName= listJL Φ

Dims used={1, 2, 3}

tallyList={40, 40, 60, 120}

{120, 120, 40, 60}

Hull # = 1

with 40 vertices

of 3D Norm = $\frac{1}{2} \sqrt{\frac{3}{2}} \varphi$

= $\frac{1}{4} \sqrt{9 - 3 \sqrt{5}}$

= 0.3785

Vertex #'s = {1, 40}

Hull # = 2

with 40 vertices

of 3D Norm = $\frac{\sqrt{\frac{3}{2}}}{2}$

= $\frac{\sqrt{\frac{3}{2}}}{2}$

= 0.6124

Vertex #'s = {41, 80}

Hull # = 3

with 60 vertices

of 3D Norm = $\frac{1}{\sqrt{2}}$

= $\frac{1}{\sqrt{2}}$

= 0.7071

Vertex #'s = {81, 140}

Hull # = 4

with 120 vertices

of 3D Norm = $\sqrt{\frac{1}{4 \varphi^2} + \frac{\varphi^4}{8}}$

= $\frac{\sqrt{13 - \sqrt{5}}}{4}$

= 0.8202

Vertex #'s = {141, 260}

Hull # = 5

with 120 vertices

of 3D Norm = $\frac{\sqrt{\frac{\tau}{2}}}{2}$

= $\frac{\sqrt{\frac{\tau}{2}}}{2}$

= 0.9354

Vertex #'s = {261, 380}

Hull # = 6

with 120 vertices

of 3D Norm = $\sqrt{\frac{1}{8 \varphi^4} + \frac{\varphi^2}{4}}$

= $\frac{\sqrt{13 + \sqrt{5}}}{4}$

= 0.9758

Vertex #'s = {381, 500}

Hull # = 7

with 40 vertices

of 3D Norm = $\frac{\sqrt{\frac{3}{2}}}{2 \varphi}$

= $\frac{1}{8} (\sqrt{6} + \sqrt{30})$

= 0.9908

Vertex #'s = {501, 540}

Hull # = 8

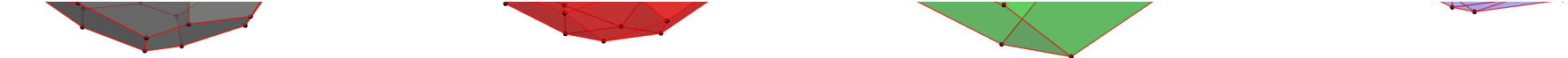
with 60 vertices

of 3D Norm = 1

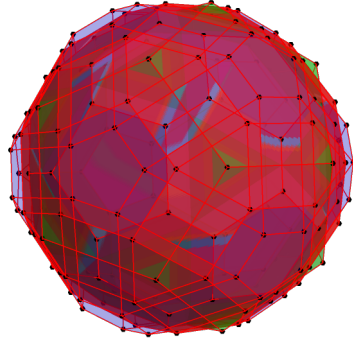
= 1

= 1.

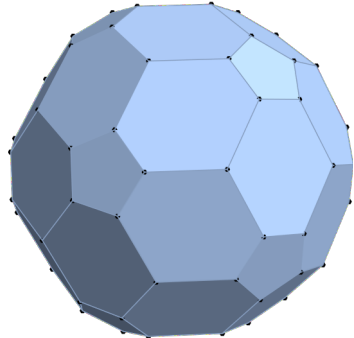
Vertex #'s = {541, 600}



Combined Hulls=



Overall Hull=



```
In[ ]:=  
  
tally3 = Sort@Tally[ {tmp = Sort@rndMat[  $\sqrt{8}$  Abs@# /.  $\varphi \rightarrow \frac{1}{2} (-1. + \sqrt{5})$  ] ;  
  
      octSym[tmp] /.  $\varphi \rightarrow 1/\varphi$ , tmp } & /@ listJL@[[All, ;; 4], #1[[2]] == #2[[2]] &] ;  
  
{Column[{Row@{#[[1, 1], "/  $\sqrt{8}$  "}, #[[1, 2]] /  $\sqrt{8}$  }, Center], #[[2]]} & /@ tally3;  
% // MatrixForm
```

Out[]//MatrixForm=

$\{0, 0, 2, 2\} / \sqrt{8}$	24
$\{0, 0, 0.707107, 0.707107\}$	
$\{0, \frac{1}{\varphi^2}, 1, \varphi^2\} / \sqrt{8}$	96
$\{0, 0.135057, 0.353553, 0.925603\}$	
$\{0, \frac{1}{\varphi}, \varphi, \sqrt{5}\} / \sqrt{8}$	96
$\{0, 0.218496, 0.572049, 0.790581\}$	
$\{1, 1, 1, \sqrt{5}\} / \sqrt{8}$	64
$\{0.353553, 0.353553, 0.353553, 0.790581\}$	
$\{\frac{1}{\varphi^2}, \varphi, \varphi, \varphi\} / \sqrt{8}$	64
$\{0.135057, 0.572049, 0.572049, 0.572049\}$	
$\{\frac{1}{\varphi}, 1, \varphi, 2\} / \sqrt{8}$	192
$\{0.218496, 0.353553, 0.572049, 0.707107\}$	
$\{\frac{1}{\varphi}, \frac{1}{\varphi}, \frac{1}{\varphi}, \varphi^2\} / \sqrt{8}$	64
$\{0.218496, 0.218496, 0.218496, 0.925603\}$	

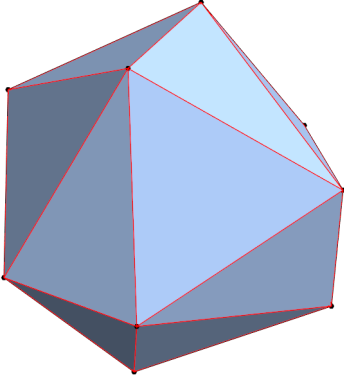
```
In[*]:=
diminishedJϕInv3 = Select[listJLϕ,
  FullSimplify[Sort@Abs[√8 #], Assumptions → {ϕ ∈ Reals, ϕ > 0}] == {0, 0, 0, 0, 0, 0, 2, 2} ||
  FullSimplify[Sort@Abs[√8 #], Assumptions → {ϕ ∈ Reals, ϕ > 0}] == {0, 0, 0, 0, 0, 1, 1/ϕ², ϕ²} &];

Length@%
hulls3DPerms["diminishedJϕInv3", False, , 1]
```

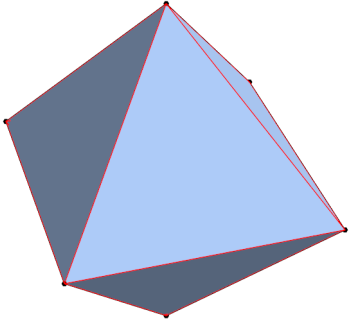
Out[*]=
120

Out[*]=
ListName= diminishedJϕInv3
(Dims used={1, 2, 3}
tallyList={24, 12, 24, 24}
{36})

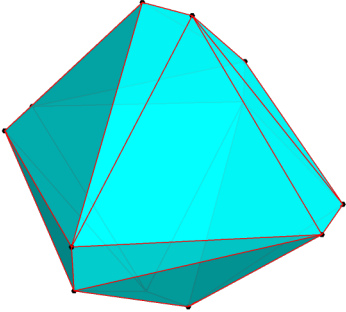
Hull # = 1
with 24 vertices
of 3D Norm = $\sqrt{\frac{1}{8} + \frac{\phi^4}{8}}$
= $\frac{1}{4} \sqrt{9 - 3 \sqrt{5}}$
= 0.3785
Vertex #'s = {1, 24}



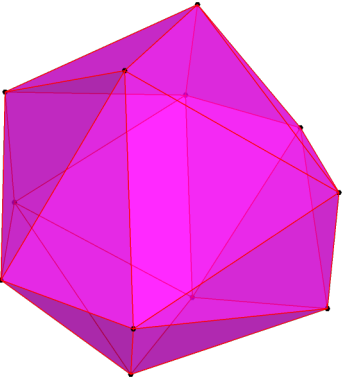
Hull # = 2
with 12 vertices
of 3D Norm = $\frac{1}{\sqrt{2}}$
= $\frac{1}{\sqrt{2}}$
= 0.7071
Vertex #'s = {25, 36}



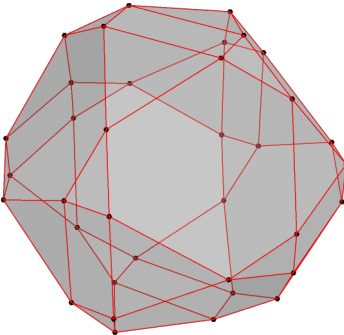
Hull # = 3
with 24 vertices
of 3D Norm = $\sqrt{\frac{1}{8 \phi^4} + \frac{|\phi^4|}{8}}$
= $\frac{\sqrt{2}}{2}$
= 0.9354
Vertex #'s = {37, 60}



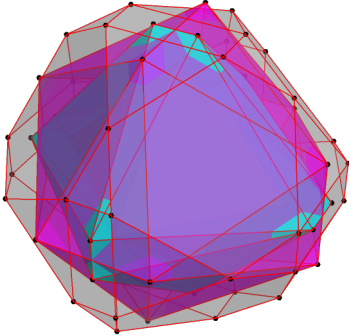
Hull # = 4
with 24 vertices
of 3D Norm = $\sqrt{\frac{1}{8} + \frac{1}{8 \phi^4}}$
= $\frac{1}{4} \sqrt{3 (3 + \sqrt{5})}$
= 0.9908
Vertex #'s = {61, 84}



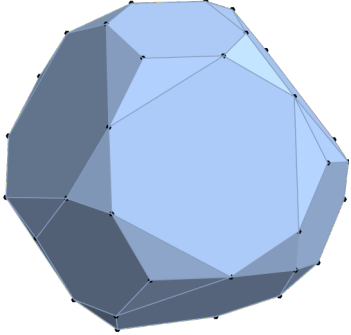
Hull # = 5
with 36 vertices
of 3D Norm = 1
= 1
= 1.
Vertex #'s = {85, 120}



Combined Hulls=



Overall Hull=



```
In[*]:=
diminishedJ3 = Select[listJL, ! MemberQ[diminishedJInv3, #] &];
Length@%
hulls3DPerms["diminishedJ3", False, , 1]
```

Out[*]//MatrixForm=

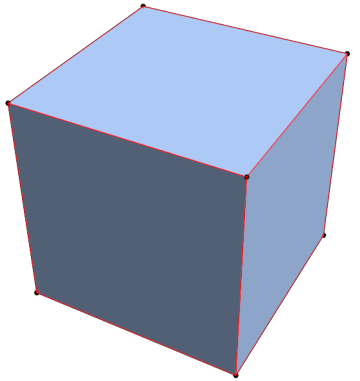
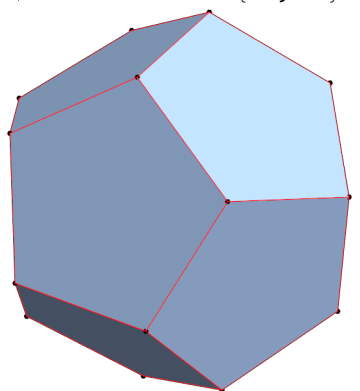
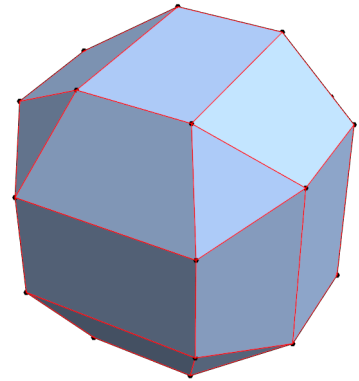
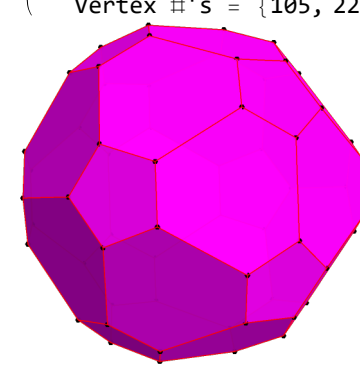
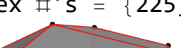
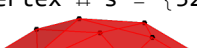

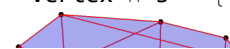
$$\left(\begin{array}{cc} \{0, 0, 2, 2\} / \sqrt{8} & 24 \\ \{0, 0, 0.707107, 0.707107\} & \\ \left\{0, \frac{1}{\varphi^2}, 1, \varphi^2\right\} / \sqrt{8} & 96 \\ \{0, 0.135057, 0.353553, 0.925603\} & \\ \left\{0, \frac{1}{\varphi}, \varphi, \sqrt{5}\right\} / \sqrt{8} & 96 \\ \{0, 0.218496, 0.572049, 0.790581\} & \\ \{1, 1, 1, \sqrt{5}\} / \sqrt{8} & 64 \\ \{0.353553, 0.353553, 0.353553, 0.790581\} & \\ \left\{\frac{1}{\varphi^2}, \varphi, \varphi, \varphi\right\} / \sqrt{8} & 64 \\ \{0.135057, 0.572049, 0.572049, 0.572049\} & \\ \left\{\frac{1}{\varphi}, 1, \varphi, 2\right\} / \sqrt{8} & 192 \\ \{0.218496, 0.353553, 0.572049, 0.707107\} & \\ \left\{\frac{1}{\varphi}, \frac{1}{\varphi}, \frac{1}{\varphi}, \varphi^2\right\} / \sqrt{8} & 64 \\ \{0.218496, 0.218496, 0.218496, 0.925603\} & \end{array} \right)$$

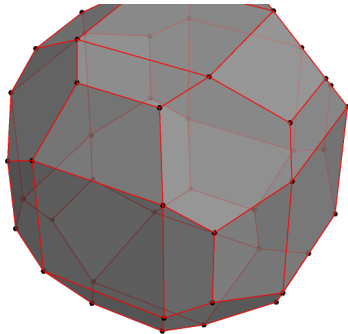
Out[*]=
480

Out[*]=

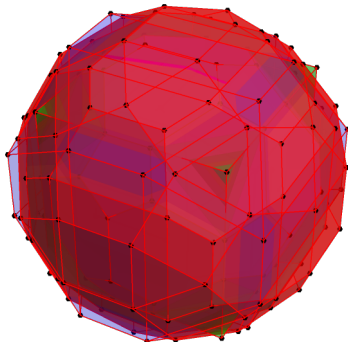
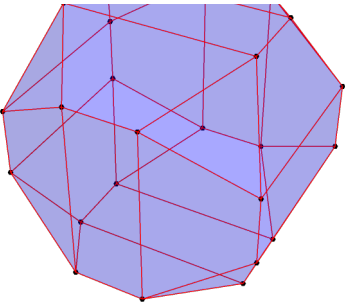
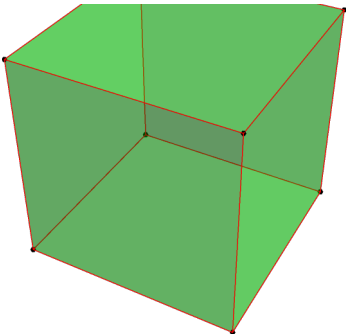
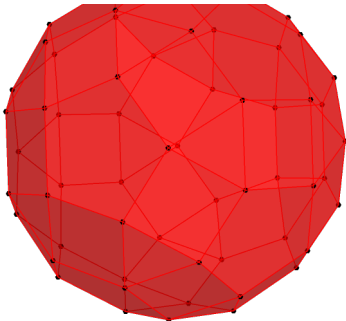
ListName= diminishedJ3

(Dims used={1, 2, 3}
tallyList={16, 40, 48, 120}
{96, 120, 16, 24}

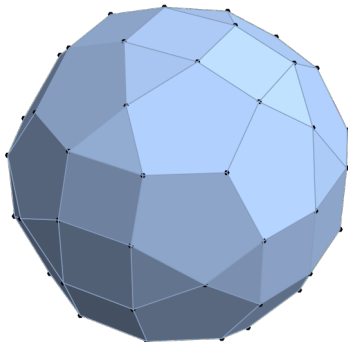
<p>Hull # = 1 with 16 vertices</p> <p>of 3D Norm = $\frac{1}{2} \sqrt{\frac{3}{2}} \varphi$ = $\frac{1}{4} \sqrt{9-3} \sqrt{5}$ = 0.3785</p> <p>Vertex #'s = {1, 16}</p> 	<p>Hull # = 2 with 40 vertices</p> <p>of 3D Norm = $\frac{\sqrt{\frac{3}{2}}}{2}$ = $\frac{\sqrt{\frac{3}{2}}}{2}$ = 0.6124</p> <p>Vertex #'s = {17, 56}</p> 	<p>Hull # = 3 with 48 vertices</p> <p>of 3D Norm = $\sqrt{\frac{1}{8} + \frac{1}{8\varphi^2} + \frac{\varphi^2}{8}}$ = $\frac{1}{\sqrt{2}}$ = 0.7071</p> <p>Vertex #'s = {57, 104}</p> 	<p>Hull # = 4 with 120 vertices</p> <p>of 3D Norm = $\sqrt{\frac{1}{4\varphi^2} + \frac{\varphi^4}{8}}$ = $\frac{\sqrt{13-\sqrt{5}}}{4}$ = 0.8202</p> <p>Vertex #'s = {105, 224}</p> 
<p>Hull # = 5 with 96 vertices</p> <p>of 3D Norm = $\frac{\sqrt{\frac{7}{2}}}{2}$ = $\frac{\sqrt{\frac{7}{2}}}{2}$ = 0.9354</p> <p>Vertex #'s = {225, 320}</p> 	<p>Hull # = 6 with 120 vertices</p> <p>of 3D Norm = $\sqrt{\frac{1}{8\varphi^4} + \frac{\varphi^2}{4}}$ = $\frac{\sqrt{13+\sqrt{5}}}{4}$ = 0.9758</p> <p>Vertex #'s = {321, 440}</p> 	<p>Hull # = 7 with 16 vertices</p> <p>of 3D Norm = $\frac{\sqrt{\frac{3}{2}}}{2\varphi}$ = $\frac{1}{8} (\sqrt{6} + \sqrt{30})$ = 0.9908</p> <p>Vertex #'s = {441, 456}</p> 	<p>Hull # = 8 with 24 vertices</p> <p>of 3D Norm = $\sqrt{\frac{5}{8} + \frac{1}{8\varphi^2} + \frac{\varphi^2}{8}}$ = 1 = 1.</p> <p>Vertex #'s = {457, 480}</p> 



Combined Hulls=



Overall Hull=



$$\ln[\bullet] :=$$

```
diminishedJInv4 = Select[listJL,
  FullSimplify[Sort@Abs[ $\sqrt{8}$  #], Assumptions  $\rightarrow \{\varphi \in \text{Reals}, \varphi > 0\}$ ] == {0, 0, 0, 0, 0, 0, 2, 2} ||
  FullSimplify[Sort@Abs[ $\sqrt{8}$  #], Assumptions  $\rightarrow \{\varphi \in \text{Reals}, \varphi > 0\}$ ] == {0, 0, 0, 0, 0,  $\sqrt{5}$ ,  $\frac{1}{\varphi}$ ,  $\varphi$  } &];
Length@%
hulls3DPerms["diminishedJInv4", False, , 1]
```

$$Out[\bullet]=$$

120

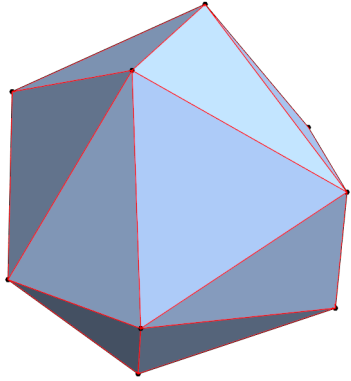
`Out[] =`

ListName= diminishedJInv4

```
( Dims used={1, 2, 3}
```

```
tallyList={24, 12, 24, 24}
```

{36}

$$\left(\begin{array}{l} \text{Hull } \sharp = 1 \\ \text{with 24 vertices} \\ \text{of 3D Norm} = \sqrt{\frac{1}{8\varphi^2} + \frac{\varphi^2}{8}} \\ = \frac{\sqrt{\frac{3}{2}}}{2} \\ = \mathbf{0.6124} \\ \text{Vertex } \sharp\text{'s} = \{1, 24\} \end{array} \right)$$


Hull # = 2

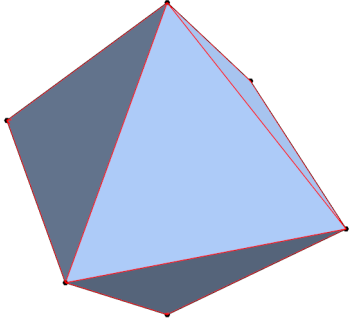
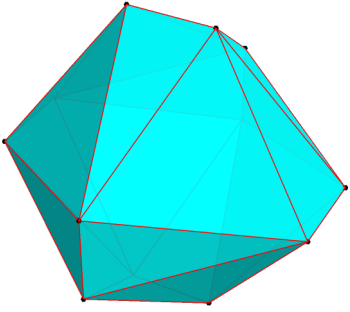
with 12 vertices

of 3D Norm = $\frac{1}{\sqrt{2}}$

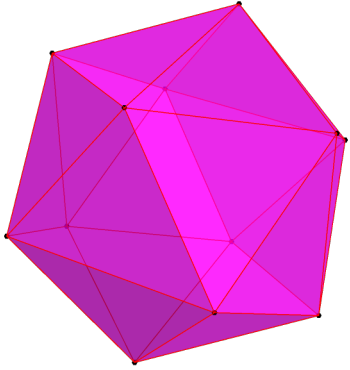
= $\frac{1}{\sqrt{2}}$

= 0.7071

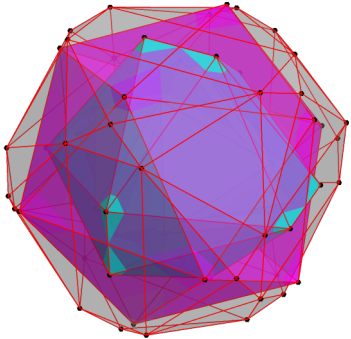
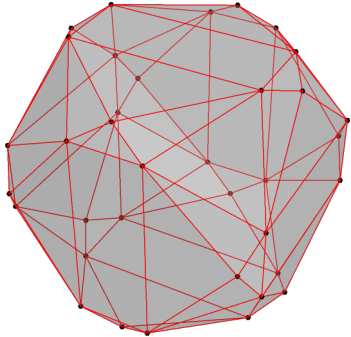
Vertex #'s = { 25, 36 }


$$\left(\begin{array}{l} \text{Hull } \# = 3 \\ \text{with 24 vertices} \\ \text{of 3D Norm} = \sqrt{\frac{5}{8} + \frac{\varphi^2}{8}} \\ = \frac{\sqrt{13 - \sqrt{5}}}{4} \\ = 0.8202 \\ \text{Vertex } \# \text{'s} = \{37, 60\} \end{array} \right)$$


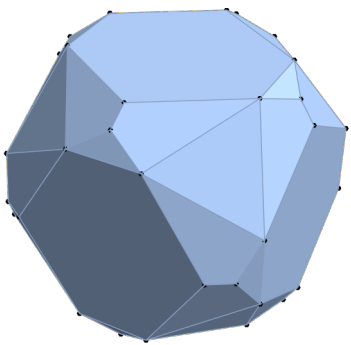
$$\left(\begin{array}{l} \text{Hull } \sharp = 4 \\ \text{with 24 vertices} \\ \text{of 3D Norm} = \sqrt{\frac{5}{8} + \frac{1}{8}\psi^2} \\ = \frac{\sqrt{13+\sqrt{5}}}{4} \\ = 0.9758 \\ \text{Vertex } \sharp\text{'s} = \{61, 84\} \end{array} \right)$$



Combined Hulls=

$$\left(\begin{array}{l} \text{Hull } \# = 5 \\ \text{with 36 vertices} \\ \text{of 3D Norm} \quad = \quad 1 \\ \quad \quad \quad = \quad 1 \\ \quad \quad \quad = \quad 1. \\ \text{Vertex } \# \text{'s} = \{85, 120\} \end{array} \right)$$


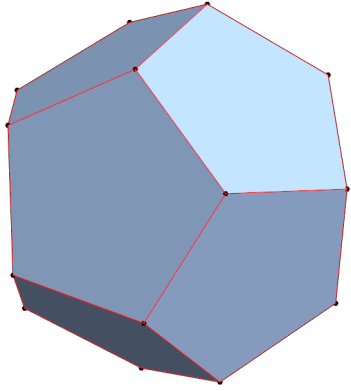
Overall Hull=

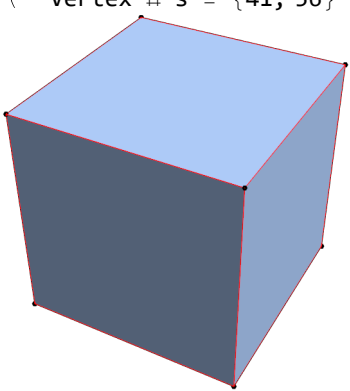


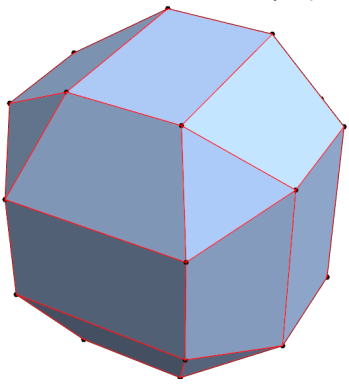
```
In[*]:=
diminishedJ4 = Select[listJL, ! MemberQ[diminishedJInv4, #] &];
Length@%
hulls3DPerms["diminishedJ4", False, , 1]
```

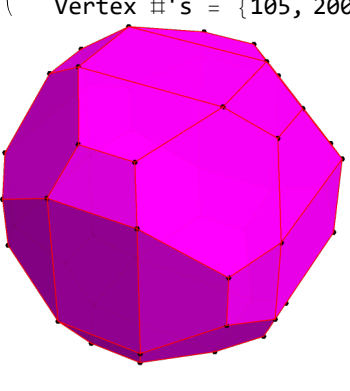
Out[*]=
480

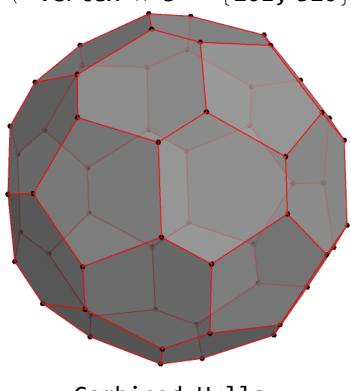
Out[*]=
ListName= diminishedJ4
(
Dims used={1, 2, 3}
tallyList={40, 16, 48, 96}
{120, 96, 40, 24}

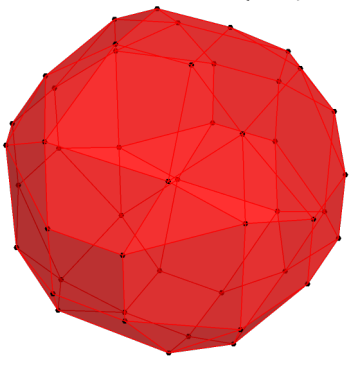
Hull # = 1
with 40 vertices
of 3D Norm = $\frac{1}{2} \sqrt{\frac{3}{2}} \varphi$
= $\frac{1}{4} \sqrt{9 - 3 \sqrt{5}}$
= 0.3785
Vertex #'s = {1, 40}


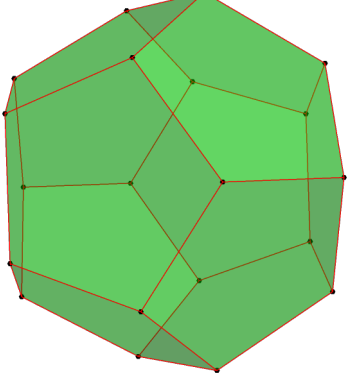
Hull # = 2
with 16 vertices
of 3D Norm = $\frac{\sqrt{\frac{3}{2}}}{2}$
= $\frac{\sqrt{\frac{3}{2}}}{2}$
= 0.6124
Vertex #'s = {41, 56}


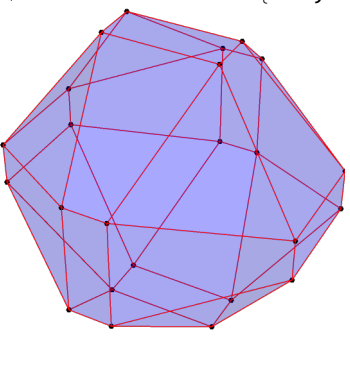
Hull # = 3
with 48 vertices
of 3D Norm = $\sqrt{\frac{1}{8} + \frac{1}{8 \varphi^2} + \frac{\varphi^2}{8}}$
= $\frac{1}{\sqrt{2}}$
= 0.7071
Vertex #'s = {57, 104}


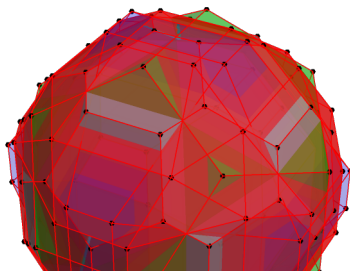
Hull # = 4
with 96 vertices
of 3D Norm = $\sqrt{\frac{1}{4 \varphi^2} + \frac{\varphi^4}{8}}$
= $\frac{\sqrt{13 - \sqrt{5}}}{4}$
= 0.8202
Vertex #'s = {105, 200}


Hull # = 5
with 120 vertices
of 3D Norm = $\frac{\sqrt{\frac{7}{2}}}{2}$
= $\frac{\sqrt{\frac{7}{2}}}{2}$
= 0.9354
Vertex #'s = {201, 320}


Hull # = 6
with 96 vertices
of 3D Norm = $\sqrt{\frac{1}{8 \varphi^4} + \frac{\varphi^2}{4}}$
= $\frac{\sqrt{13 + \sqrt{5}}}{4}$
= 0.9758
Vertex #'s = {321, 416}


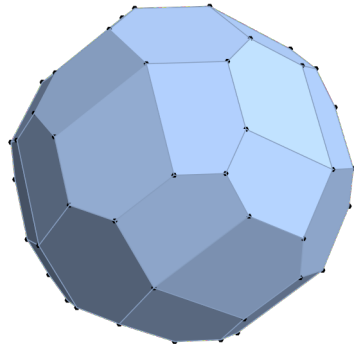
Hull # = 7
with 40 vertices
of 3D Norm = $\frac{\sqrt{\frac{3}{2}}}{2 \varphi}$
= $\frac{1}{8} (\sqrt{6} + \sqrt{30})$
= 0.9908
Vertex #'s = {417, 456}


Hull # = 8
with 24 vertices
of 3D Norm = $\sqrt{\frac{1}{8} + \frac{1}{8 \varphi^4} + \frac{\varphi^4}{8}}$
= 1
= 1.
Vertex #'s = {457, 480}


Combined Hulls=




Overall Hull=




```
In[*]:=
diminishedJInv5 = Select[listJL,
  (*) FullSimplify[Sort@Abs[√8 #], Assumptions → {φ ∈ Reals, φ > 0}] == {0, 0, 0, 0, 0, 0, 2, 2} || **)
  FullSimplify[Sort@Abs[√8 #], Assumptions → {φ ∈ Reals, φ > 0}] == {0, 0, 0, 0, 1, 2, 1/φ, φ} &];

Length@%
hulls3DPerms["diminishedJInv5", False, , 1]
```

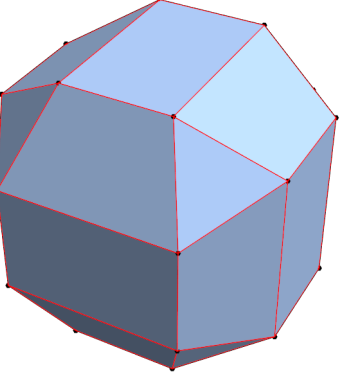
```
Out[*]=
192

Out[*]=
ListName= diminishedJInv5
Dims used={1, 2, 3}
tallyList={48, 48, 48, 48}
```

Hull # = 1
with 48 vertices

of 3D Norm = $\sqrt{\frac{1}{8} + \frac{1}{8\varphi^2} + \frac{\varphi^2}{8}}$
= $\frac{1}{\sqrt{2}}$
= 0.7071

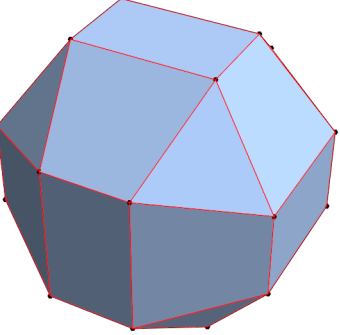
Vertex #'s = {1, 48}



Hull # = 2
with 48 vertices

of 3D Norm = $\sqrt{\frac{5}{8} + \frac{\varphi^2}{8}}$
= $\frac{\sqrt{13-\sqrt{5}}}{4}$
= 0.8202

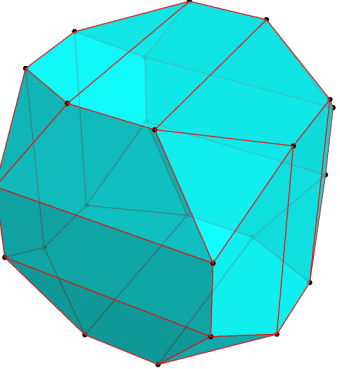
Vertex #'s = {49, 96}



Hull # = 3
with 48 vertices

of 3D Norm = $\sqrt{\frac{1}{2} + \frac{1}{8\varphi^2} + \frac{\varphi^2}{8}}$
= $\frac{\sqrt{\frac{7}{2}}}{2}$
= 0.9354

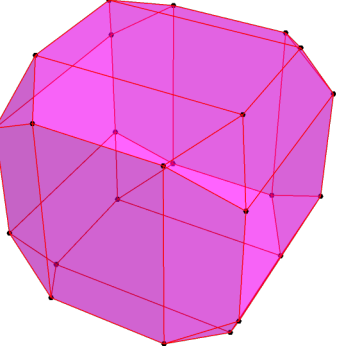
Vertex #'s = {97, 144}



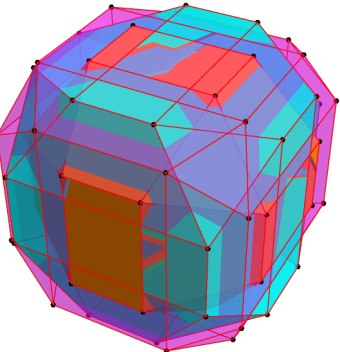
Hull # = 4
with 48 vertices

of 3D Norm = $\sqrt{\frac{5}{8} + \frac{1}{8\varphi^2}}$
= $\frac{\sqrt{13+\sqrt{5}}}{4}$
= 0.9758

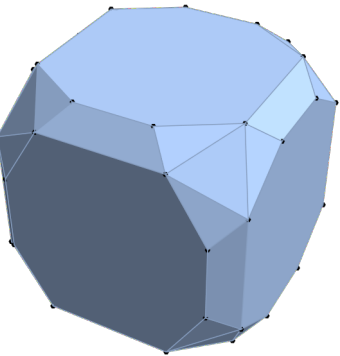
Vertex #'s = {145, 192}



Combined Hulls=



Overall Hull=



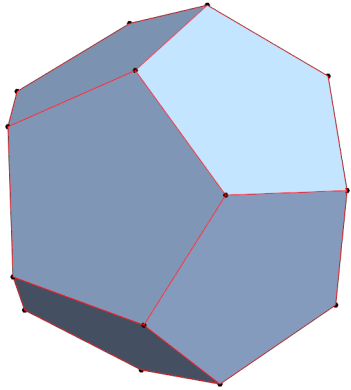
```
In[*]:=
diminishedJ5 = Select[listJL, ! MemberQ[diminishedJInv5, #] &];
Length@%
hulls3DPerms["diminishedJ5", False, , 1]
```

Out[*]=
408

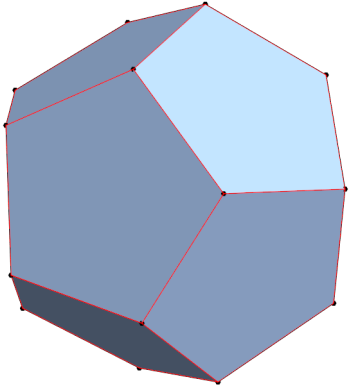
Out[*]=
ListName= diminishedJ5

(Dims used={1, 2, 3}
tallyList={40, 40, 12, 72}
 {72, 72, 40, 60}

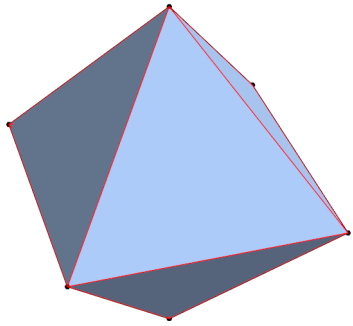
$$\left(\begin{array}{l} \text{Hull } \# = 1 \\ \text{with } 40 \text{ vertices} \\ \text{of 3D Norm} = \frac{1}{2} \sqrt{\frac{3}{2}} \varphi \\ = \frac{1}{4} \sqrt{9 - 3 \sqrt{5}} \\ = 0.3785 \\ \text{Vertex } \#'s = \{1, 40\} \end{array} \right)$$



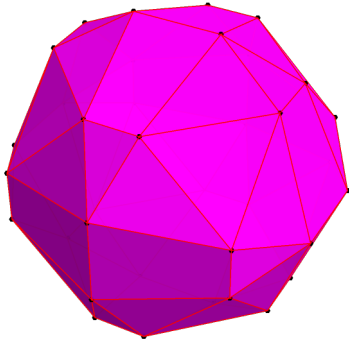
$$\left(\begin{array}{l} \text{Hull } \# = 2 \\ \text{with } 40 \text{ vertices} \\ \text{of 3D Norm} = \frac{\sqrt{\frac{3}{2}}}{2} \\ = \frac{\sqrt{\frac{3}{2}}}{2} \\ = 0.6124 \\ \text{Vertex } \#'s = \{41, 80\} \end{array} \right)$$



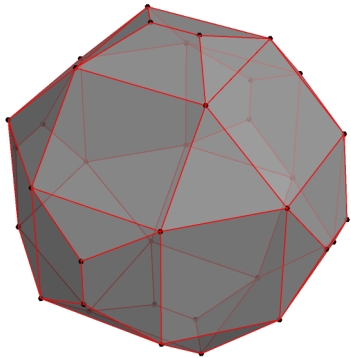
$$\left(\begin{array}{l} \text{Hull } \# = 3 \\ \text{with } 12 \text{ vertices} \\ \text{of 3D Norm} = \frac{1}{\sqrt{2}} \\ = \frac{1}{\sqrt{2}} \\ = 0.7071 \\ \text{Vertex } \#'s = \{81, 92\} \end{array} \right)$$



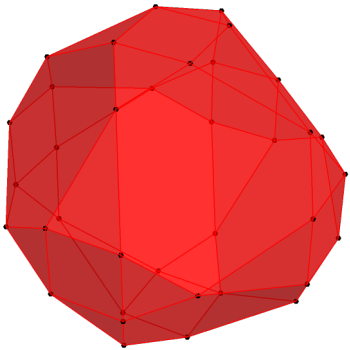
$$\left(\begin{array}{l} \text{Hull } \# = 4 \\ \text{with } 72 \text{ vertices} \\ \text{of 3D Norm} = \sqrt{\frac{1}{4 \varphi^2} + \frac{|\varphi|^4}{8}} \\ = \frac{\sqrt{13 - \sqrt{5}}}{4} \\ = 0.8202 \\ \text{Vertex } \#'s = \{93, 164\} \end{array} \right)$$



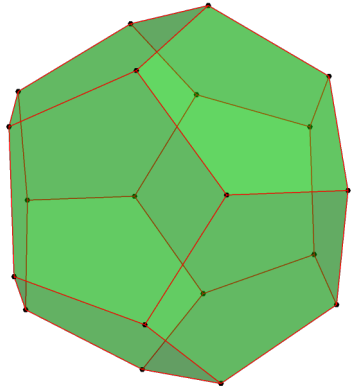
$$\left(\begin{array}{l} \text{Hull } \# = 5 \\ \text{with } 72 \text{ vertices} \\ \text{of 3D Norm} = \frac{\sqrt{\frac{7}{2}}}{2} \\ = \frac{\sqrt{\frac{7}{2}}}{2} \\ = 0.9354 \\ \text{Vertex } \#'s = \{165, 236\} \end{array} \right)$$



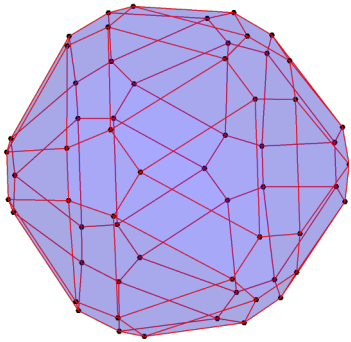
$$\left(\begin{array}{l} \text{Hull } \# = 6 \\ \text{with } 72 \text{ vertices} \\ \text{of 3D Norm} = \sqrt{\frac{1}{8 \varphi^4} + \frac{|\varphi|^2}{4}} \\ = \frac{\sqrt{13 + \sqrt{5}}}{4} \\ = 0.9758 \\ \text{Vertex } \#'s = \{237, 308\} \end{array} \right)$$



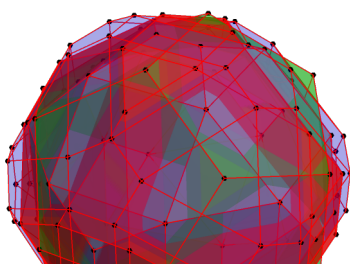
$$\left(\begin{array}{l} \text{Hull } \# = 7 \\ \text{with } 40 \text{ vertices} \\ \text{of 3D Norm} = \frac{\sqrt{\frac{3}{2}}}{2 \varphi} \\ = \frac{1}{8} (\sqrt{6} + \sqrt{30}) \\ = 0.9908 \\ \text{Vertex } \#'s = \{309, 348\} \end{array} \right)$$

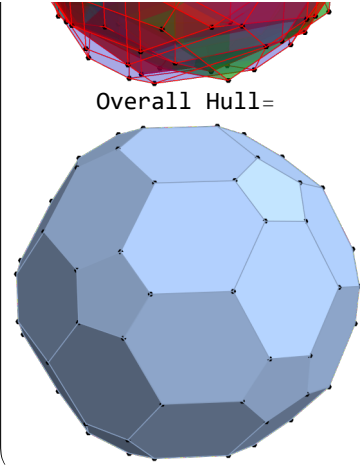


$$\left(\begin{array}{l} \text{Hull } \# = 8 \\ \text{with } 60 \text{ vertices} \\ \text{of 3D Norm} = 1 \\ = 1 \\ = 1. \\ \text{Vertex } \#'s = \{349, 408\} \end{array} \right)$$



Combined Hulls=





```
In[*]:=
diminishedJInv6 = Select[listJL,
  (*) FullSimplify[Sort@Abs[ Sqrt[8] #], Assumptions -> {φ ∈ Reals, φ > 0}] == {0, 0, 0, 0, 0, 2, 2} || **
  FullSimplify[Sort@Abs[ Sqrt[8] #], Assumptions -> {φ ∈ Reals, φ > 0}] == {0, 0, 0, 0, 1, 1, 1, Sqrt[5]} ||
  FullSimplify[Sort@Abs[ Sqrt[8] #], Assumptions -> {φ ∈ Reals, φ > 0}] == {0, 0, 0, 0, 1/φ, 1/φ, 1/φ, φ^2} ||
  FullSimplify[Sort@Abs[ Sqrt[8] #], Assumptions -> {φ ∈ Reals, φ > 0}] == {0, 0, 0, 0, 1/φ^2, φ, φ, φ} &];

Length@%
hulls3DPerms["diminishedJInv6", False, , 1]
```

Out[*]=

192

Out[]=

ListName= diminishedJInv6

Dims used={1, 2, 3}

tallyList={16, 16, 48, 48}

{48, 16}

Hull # = 1

with 16 vertices

of 3D Norm = $\frac{1}{2} \sqrt{\frac{3}{2}} \varphi$

= $\frac{1}{4} \sqrt{9 - 3 \sqrt{5}}$

= 0.3785

Vertex #'s = {1, 16}

Hull # = 2

with 16 vertices

of 3D Norm = $\frac{\sqrt{\frac{3}{2}}}{2}$

= $\frac{\sqrt{\frac{3}{2}}}{2}$

= 0.6124

Vertex #'s = {17, 32}

Hull # = 3

with 48 vertices

of 3D Norm = $\sqrt{\frac{1}{4 \varphi^2} + \frac{\varphi^4}{8}}$

= $\frac{\sqrt{13 - \sqrt{5}}}{4}$

= 0.8202

Vertex #'s = {33, 80}

Hull # = 4

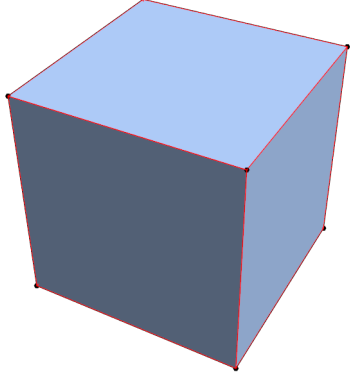
with 48 vertices

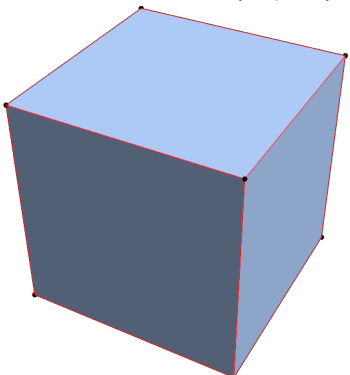
of 3D Norm = $\frac{\sqrt{\frac{7}{2}}}{2}$

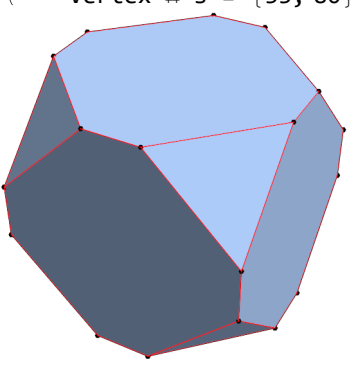
= $\frac{\sqrt{\frac{7}{2}}}{2}$

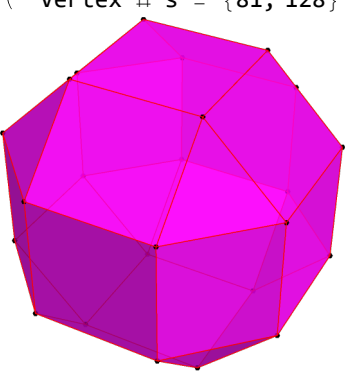
= 0.9354

Vertex #'s = {81, 128}









Hull # = 5

with 48 vertices

of 3D Norm = $\sqrt{\frac{1}{8 \varphi^4} + \frac{\varphi^2}{4}}$

= $\frac{\sqrt{13 + \sqrt{5}}}{4}$

= 0.9758

Vertex #'s = {129, 176}

Hull # = 6

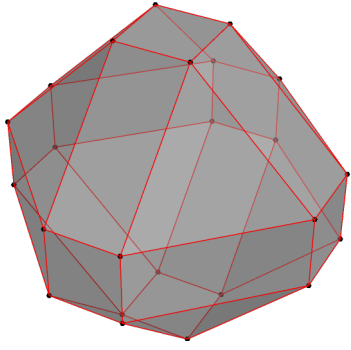
with 16 vertices

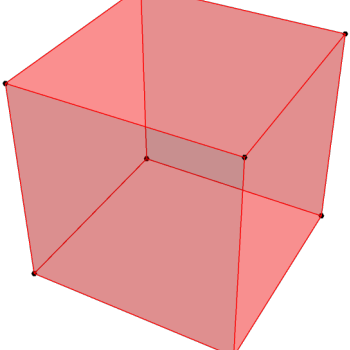
of 3D Norm = $\frac{\sqrt{\frac{3}{2}}}{2 \varphi}$

= $\frac{1}{8} (\sqrt{6} + \sqrt{30})$

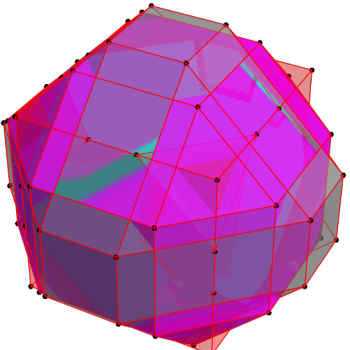
= 0.9908

Vertex #'s = {177, 192}

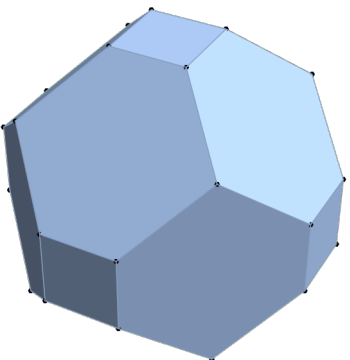




Combined Hulls=



Overall Hull=



In[]:=

diminishedJ6 = Select[listJL, ! MemberQ[diminishedJInv6, #] &];

Length@%

hulls3DPerms["diminishedJ6", False, , 1]

Out[]=

408

Out[]=

ListName= diminishedJ6

Dims used={1, 2, 3}

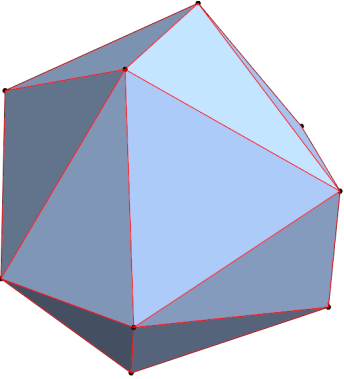
tallyList={24, 24, 60, 72}

{72, 72, 24, 60}

Hull # = 1
with 24 vertices

of 3D Norm = $\sqrt{\frac{1}{8} + \frac{\varphi^4}{8}}$
= $\frac{1}{4} \sqrt{9 - 3 \sqrt{5}}$
= 0.3785

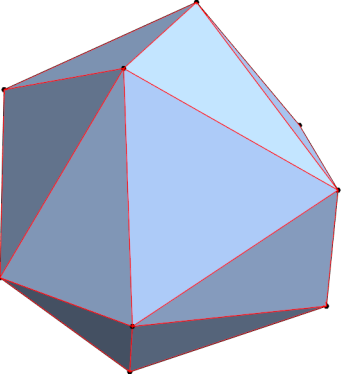
Vertex #'s = {1, 24}



Hull # = 2
with 24 vertices

of 3D Norm = $\sqrt{\frac{1}{8 \varphi^2} + \frac{\varphi^2}{8}}$
= $\frac{\sqrt{\frac{3}{2}}}{2}$
= 0.6124

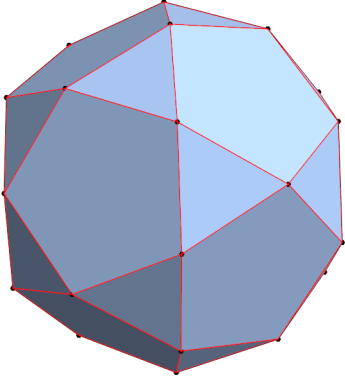
Vertex #'s = {25, 48}



Hull # = 3
with 60 vertices

of 3D Norm = $\frac{1}{\sqrt{2}}$
= $\frac{1}{\sqrt{2}}$
= 0.7071

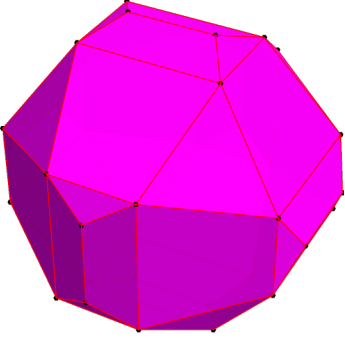
Vertex #'s = {49, 108}



Hull # = 4
with 72 vertices

of 3D Norm = $\sqrt{\frac{5}{8} + \frac{\varphi^2}{8}}$
= $\frac{\sqrt{13 - \sqrt{5}}}{4}$
= 0.8202

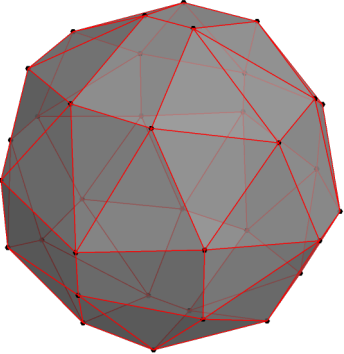
Vertex #'s = {109, 180}



Hull # = 5
with 72 vertices

of 3D Norm = $\sqrt{\frac{1}{2} + \frac{1}{8 \varphi^2} + \frac{\varphi^2}{8}}$
= $\frac{\sqrt{\frac{7}{2}}}{2}$
= 0.9354

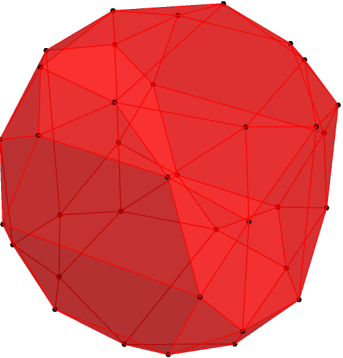
Vertex #'s = {181, 252}



Hull # = 6
with 72 vertices

of 3D Norm = $\sqrt{\frac{5}{8} + \frac{1}{8 \varphi^2}}$
= $\frac{\sqrt{13 + \sqrt{5}}}{4}$
= 0.9758

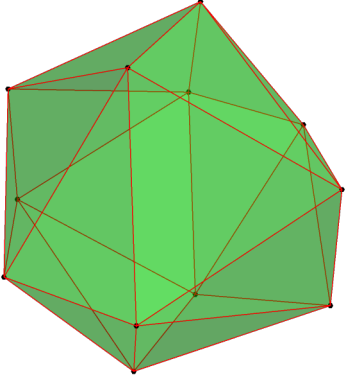
Vertex #'s = {253, 324}



Hull # = 7
with 24 vertices

of 3D Norm = $\sqrt{\frac{1}{8} + \frac{1}{8 \varphi^4}}$
= $\frac{1}{4} \sqrt{3 (3 + \sqrt{5})}$
= 0.9908

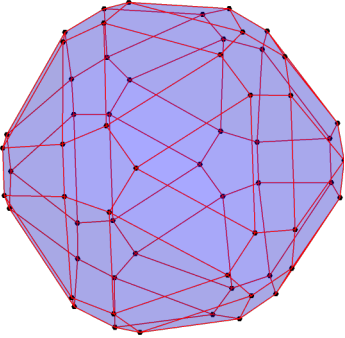
Vertex #'s = {325, 348}



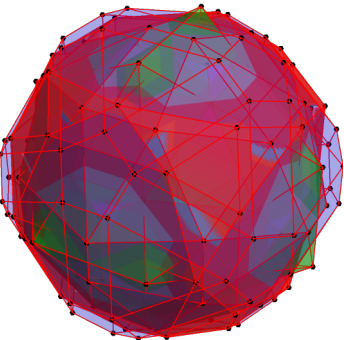
Hull # = 8
with 60 vertices

of 3D Norm = 1
= 1
= 1.

Vertex #'s = {349, 408}



Combined Hulls=



Overall Hull=

